Mach Effect Thrusters (Mets) And "Over-Unity" Energy Production

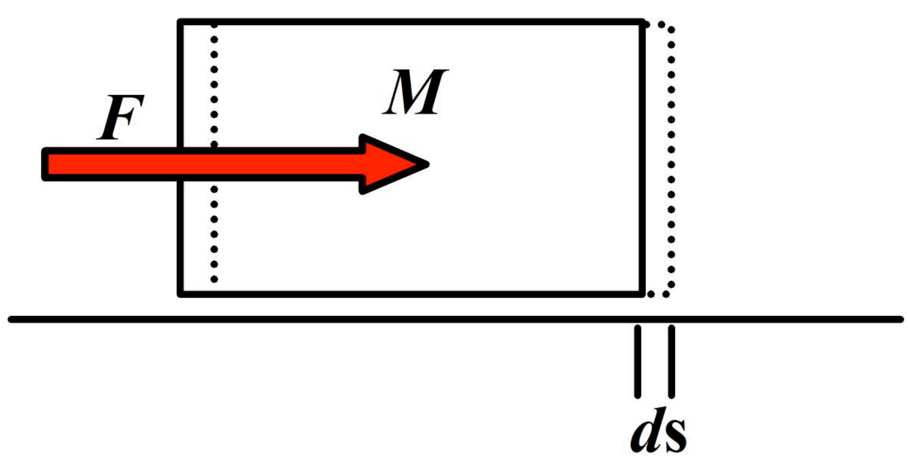
Professor Emeritus Jim Woodward
CalState Fullerton, Dept. of Physics

13 November, 2015

We routinely hear a criticism of METs based upon an argument that claims: *if a MET is operated at constant power input for a sufficiently long time, it will acquire enough kinetic energy to exceed the total input energy of operation.* Assuming this argument to be correct, critcs assert that METs violate energy conservation as the ratio of the acquired kinetic energy to total input energy exceeds "unity."

Contrary to this "over-unity" assumption, this argument is based on flawed physics and, consequently, wrong. The fact that the argument applies to **all** simple mechanical systems (in addition to METs) should have alerted critics to their mistake. But it didn't. So, a dumb idea that should have been quickly buried is still with us. The purpose of this essay is to carry out a long overdue burial.

In brief, the "over-unity" argument asserts that a constant input power into a MET will produce a constant thrust (force). This, in turn, produces a constant acceleration of any object to which the MET is attached. The constant acceleration produces a linearly increasing velocity of the object. The kinetic energy of the object, however, increases as the square of the velocity. This means that at some point, the kinetic energy of the object will exceed the total input energy used to produce the thrust as that only increases linearly with time. Critics then claimed that this purported behavior constituted violation of energy conservation and proposed it as a fatal critique of Mach Effect thrusters. Note, however, that the argument applies to **all** systems where a constant thrust produced by a constant input power produces motion.



Consider a block of mass M at rest on a smooth, level, frictionless surface. At time t_o a force $\mathbf{F}$ is applied to the block. The force is assumed constant. In an interval dt the block accelerates according to Newton's second law:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = M\mathbf{a} = M\frac{d\mathbf{v}}{dt} \tag{1}$$

Where $\mathbf{p}$ is the momentum of the block and $\mathbf{a}$ its instantaneous acceleration. $\mathbf{v}$ is the velocity of the block with respect to some chosen frame of reference. Since M and $\mathbf{F}$ are assumed constant, there is no $\mathbf{v}\ dM/dt$ term in the derivative of $\mathbf{p}$, and $\mathbf{a}$ is a constant too. This makes $\mathbf{v}$ a linearly increasing function time:

$$\mathbf{v}(t) = \mathbf{a}t + \mathbf{v}_o \tag{2}$$

where $\mathbf{a}$ is the acceleration of the block due to the application of the force. Since in our simple case of the block $\mathbf{F}$ and $\mathbf{v}$ are always in the same direction, we can drop the vector notation and simply write:

$$v(t) = at + v_o \tag{3}$$

A further simplification is possible if we assume that $v_o = 0$, so:

$$v(t) = at \tag{4}$$

To address issues involving energy, we need a definition of the relationship between "work", energy, and motion. That definition is:

$$work \equiv \mathbf{F} \bullet d\mathbf{s} = dE \tag{5}$$

Where $d\mathbf{s}$ is a small (differential) increment of distance through which the component of the force $\mathbf{F}$ in the direction of $d\mathbf{s}$ acts producing a small (differential) change in the energy of the block dE.

The issue of interest here is the evolution of the system and the power – defined as the time rate of change of energy – involved. To explore this we differentiate the work equation with respect to time. Since F is assumed constant, we get:

$$\frac{dE}{dt} = \mathbf{F} \bullet \frac{d\mathbf{s}}{dt} = \mathbf{F} \bullet \mathbf{v} \tag{6}$$

Multiplying through by dt, we get:

$$dE = \mathbf{F} \bullet d\mathbf{s} = \mathbf{F} \bullet \mathbf{v}\, dt \tag{7}$$

To get the total energy acquired during an interval of the application of a constant force, we simply integrate Equation (5):

$$\int_{E_{ii}}^{E_f} dE = \int_{t_i}^{t_f} \mathbf{F} \bullet \mathbf{v}\, dt \tag{8}$$

where the subscripts *i* and *f* denote initial and final. As noted above, in this case **v** is a linear function of time, so we substitute from Equation (4) for **v** in Equation (8) and we further assume that **F** and **v** are in the same direction so that we can ignore the dot product.

$$E_f - E_i = \int_{t_i}^{t_f} \mathbf{F} \bullet \mathbf{v}\, dt = Fa \int_{t_i}^{t_f} t\, dt = \frac{Fa}{2}\left(t_f^2 - t_i^2\right) = \frac{Fa}{2} t^2 \quad \text{if } t_{\mathrm{i}} = 0 \tag{9}$$

We see immediately that the action of a constant force on the block causes it to acquire kinetic energy that depends quadratically on the time elapsed from the inception of the action of the force. This is true for all mechanical systems where a constant force is applied to some object that is free to move under the action of the force. It is **not** a distinctive feature of the operation of METs.

So far this is all just elementary mechanics. We have not yet done anything stupid or wrong (or both). As long as we don't mess with the math, we're OK (and energy conservation is not violated). How then do some argue that in this simple system – and METs in particular – energy conservation is violated?

Simple. By doing something stupid and wrong. In particular, by taking the "figure of merit" of a thrust (force) generator – by definition, the number of Newtons of thrust produced per watt of input power to the thrust generator – and treating it as a dynamical equation that can be used to calculate the energy input to a motor that acts for some length of time; that is:

$$F_m \equiv \frac{F}{P} \tag{10}$$

where F_{m} is the figure of merit and *P* the input power to the motor that produces the thrust *F*. You might think that we can rearrange Equation (10) as follows:

$$P = \frac{dE}{dt} = \frac{F}{F_m} \tag{11}$$

This can be rearranged to:

$$dE = \frac{F}{F_m} dt \tag{12}$$

Since F and F_m are both constants, Equation (12) integrates to:

$$E_f - E_i = \frac{F}{F_m}\left(t_f - t_i\right) = \frac{F}{F_m} t \quad \text{if } t_i = 0 \tag{13}$$

Since both Equations (9) and (13) give the difference between the final and initial energies of the object on which the motor is mounted, it might seem that we can simply equate the expressions for these energy differences. That is:

$$\frac{F}{F_m} t = \frac{F a}{2} t^2 \tag{14}$$

Now we have done something stupid and wrong. This amounts to the assertion that:

$$t = \frac{F_m a}{2} t^2 \tag{15}$$

which is *obviously* wrong. For some values of t, the coefficient of t^2 on the right hand side of Equation (15) (a constant by the way) may make this equation valid. [That is, it can be treated as a simple quadratic equation and solved by the usual techniques.] As a continuous evolution equation, however, it is nonsense. But this is the mathematics of those who make the "over unity" energy conservation violation argument about the operation of METs. The real question here is how could anyone, having done this calculation or its equivalent, think that they had made a profound discovery about anything? [Or METs in particular?] After all, it is universally known that energy conservation is not violated in classical mechanics.

I won't attempt an answer to the foregoing rhetorical question. But it is worth pointing out the likely source of the error. It's the velocity. In general, velocity is not an invariant quantity as it depends on the motion of the observer as well as any velocity ascribed to motion in some other reference frame. The principle of relativity precludes singling out any particular observer as privileged over all others. The freedom of choice of reference frame is a double-edged sword. It allows you to choose a frame in which calculations can be significantly simplified because some velocities are zero for example. But it can also be troublesome as it can lead to arguments based on velocities in one frame that are very different in other frames. In the case of the "over-unity" argument, this means that an accelerating force can be producing over-unity behavior in one frame of reference, and not be doing this in another reference frame. Elementary physical intuition tells you that it should be one or the other in **all** inertial frames of reference.

An example of an analogous case that routinely comes up in elementary mechanics is the "rocket" equation. That is, the application of Newton's second law to the case of a rocket. In this case, Equation (1) reads:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = M\frac{d\mathbf{v}}{dt} + \mathbf{v}\frac{dM}{dt} = M\mathbf{a} + \mathbf{v}\frac{dM}{dt} \tag{16}$$

The second term on the right hand side – that quantifies the magnitude of the instantaneous momentum flux in the exhaust plume – is **required** to balance the changing momentum produced by the force of the propellant on the rocket's combustion chamber as it is burned. That is, $M\mathbf{a}$. $\mathbf{v}$ is the velocity of the just ejected exhaust plume with respect to the rocket motor.

Given the form of Newton's second law as stated in Equations (1) and (16), even competent physicists have come to believe that $\mathbf{v}\, dM/dt$ is a force, just as $M\mathbf{a}$ is a force. But $\mathbf{v}\, dM/dt$ isn't like an $M\mathbf{a}$ force. This is usually illustrated in elementary physics texts with problems/examples like: a railway car moves along a smooth, level, straight, frictionless track with constant velocity. A pile of sand on the bed of the car is allowed to fall through a hole in the floor of the car. Does the speed of the car relative to the Earth (which can be taken to have effectively infinite mass) change as the sand falls? A colleague who monitors the pedagogical literature tells me that people routinely mess this up – and that at intervals of five to ten years, articles or blog comments addressing this issue routinely appear. And, alas, that even those attuned to the subtleties of the issue make mistakes in handling it.

In the case of a rocket motor, the thing to observe is that there is one invariant velocity involved: that of the exhaust plume with respect to the motor. All observers, irrespective of their own motions, agree on both the magnitude and direction of this velocity. And it is the velocity that yields momentum conservation. An argument based on an incorrect application of Newton's second law to METs was advanced as a criticism of Mach Effects by an Oak Ridge scientist many years ago. It is dealt with on pages 77 and 127 of *Making Starships and Stargates: the Science of Interstellar Propulsion and Absurdly Benign Wormholes.* It will not be discussed further here.

To wrap this up, we ask: is it possible to do a correct calculation of the sort that critics did that does not lead to wrong predictions of the violation of energy conservation? By paying attention to the physics of the situation, yes, such a calculation is possible. We take Equations (9) and (13) as the integrations for the constant force work equation and the figure of merit equation respectively. We know that, starting from $t = 0$, if we let the integration interval t get very large, the work equation integral will first equal and then exceed the energy calculated by the figure of merit equation. So we require that t be sufficiently small that this obvious violation of energy conservation does not happen. Should all of the input power be transformed into kinetic energy, we would choose the positive root of the solution of Equation (15). If some of the power ends up as, for example, heat, then a smaller value of t would obtain. We then choose the value of t for the time differential that for **all** intervals to be summed to get the energies for the two methods. That is, we note what should be obvious physics for this situation: the energies added to the two sums in every differential time interval are **always** in the same ratio as they are in the very first interval because the only invariant velocity that exists in this case is the one of instantaneous rest at the outset of each interval. If this prescription – the only one that makes physical sense in the circumstances – is followed, no energy conservation violation follows from the calculation. And elementary mechanics is not threatened by an obviously wrong calculation.