An Epitaxial Device for Dynamic Interaction with the Vacuum State*

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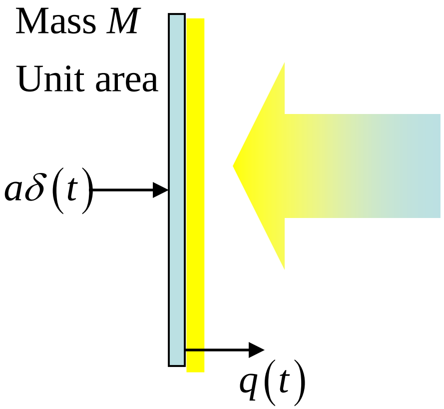
*U.S. and International patents pending

Background

- Over 60 years ago, H. B. G. Casimir and D. Polder [1, 2] explained the retarded van der Waals force in terms of the zero-point energy of a quantized field.
- Regarding the pressure on moving mirrors due to the dynamic Casimir effect, Neto and colleagues, [3-7], took a perturbative approach on the assumption that the mirror motion is << than the wavelengths of interest. (causality issues?)
- Maclay and Forward, [8], used this work to investigate the Dynamic Casimir effect as a propulsive mechanism.
  - Due to the high frequencies of mirror motion needed, they concluded that owing to the limited strength of materials, the maximum amplitudes must be at the nanometer scale.
- Recent progress (including other work presented at this workshop!) has provided experimental support
- This presentation describes an idea to attain large amplitudes, and develops analysis to support manufacture of a test item.

First Order Perturbation
Very small motion, no wavelength dependence



$$\text{Force per unit area } \cong -\frac{\hbar c}{30\pi^2}\frac{d^5 q(t)}{c^5 dt^5}$$

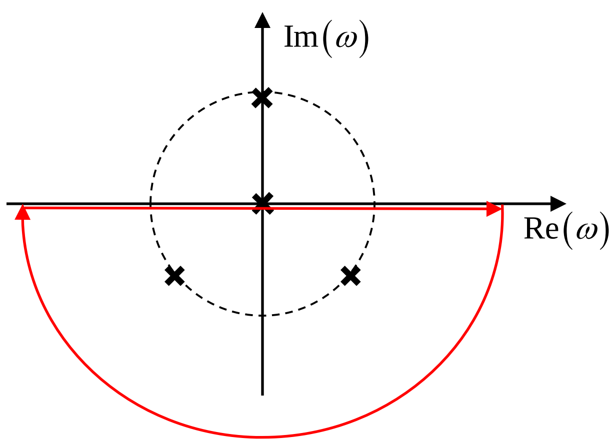
$$\downarrow$$

$$M\frac{d^2 q}{dt^2} + \frac{\hbar c}{30\pi^2}\frac{d^5 q}{c^5 dt^5} = a\delta(t)$$

Normalized e.o.m.: $\dfrac{d^2 x}{d\tau^2} + \dfrac{d^5 x}{d\tau^5} = \delta(\tau) \Rightarrow X(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} x(\tau) e^{-i\omega\tau} d\tau = \dfrac{1}{\omega^2\left(i\omega^3 - 1\right)}$

Now consider the impulse response obtained via the inverse transform of $X(\omega)$: $x(\tau) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} X(\omega) e^{i\omega\tau}$

For t<0, must take the contour in the lower half plane:

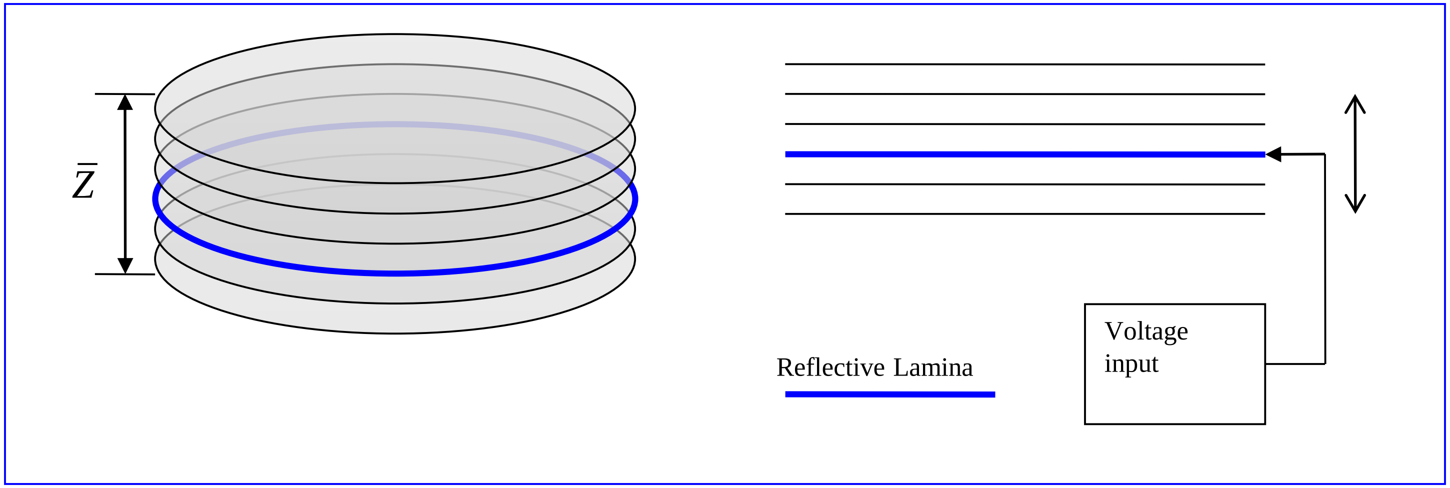


We require that $x(\tau < 0)$ vanish, for causality.

Hence there must be no poles in the lower half plane.

A moving "mirror" is a front of reflectivity – Mechanical motion unnecessary!

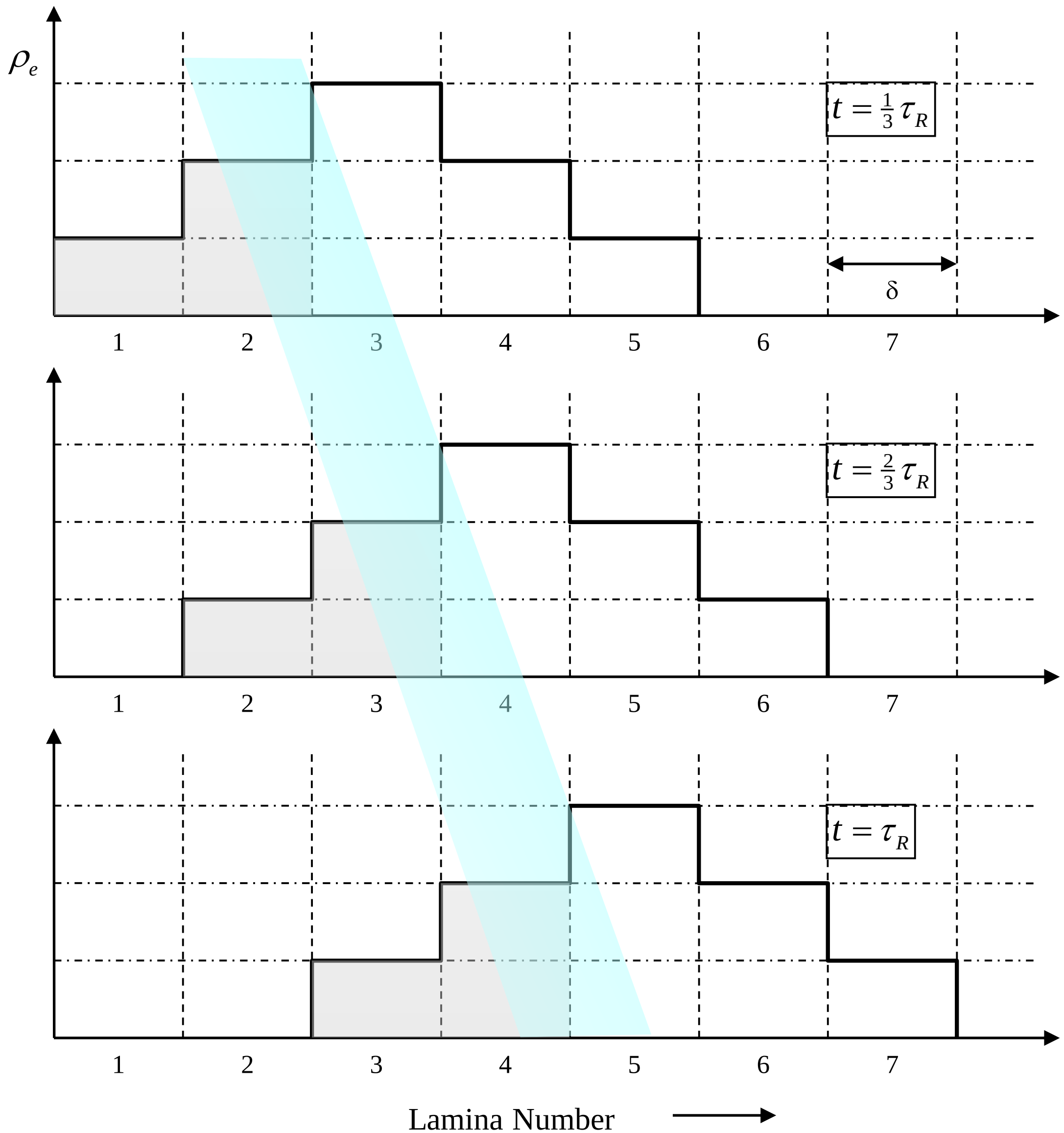
- The Casimir effect is due to the motion of the boundary conditions constraining the free field.
- The advent of transparent electrochromic semiconductors used for thin film applications, or Chiral Nematic Liquid Crystals [9-14] suggests the possibility of achieving large motions of reflective surfaces with no mechanically moving parts.
- This paper proposes the use of an epitaxial assembly of switchable laminae. **To evaluate the forces, we must consider large motion**
- **An objective is to formulate specs for manufacture of a test item**



Without voltage input, a lamina is a dielectric
With input, the lamina becomes a conductor (or vice-versa)
Inputs can be switched at high speed

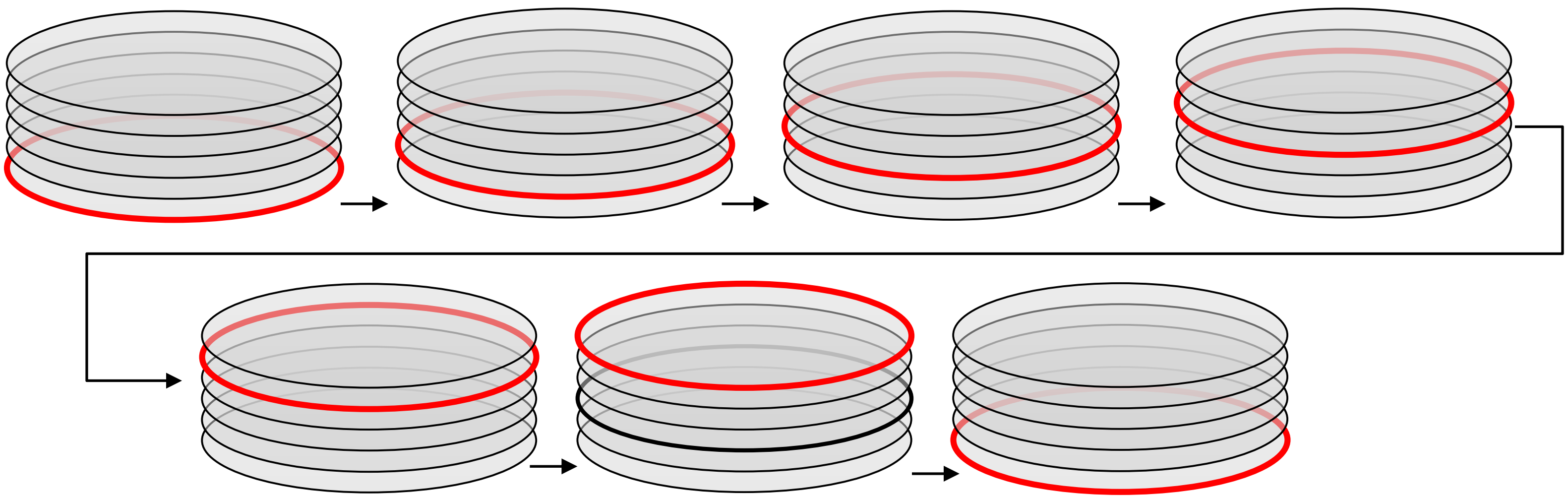
Progression of reflectivity as the laminas are successively pulsed.

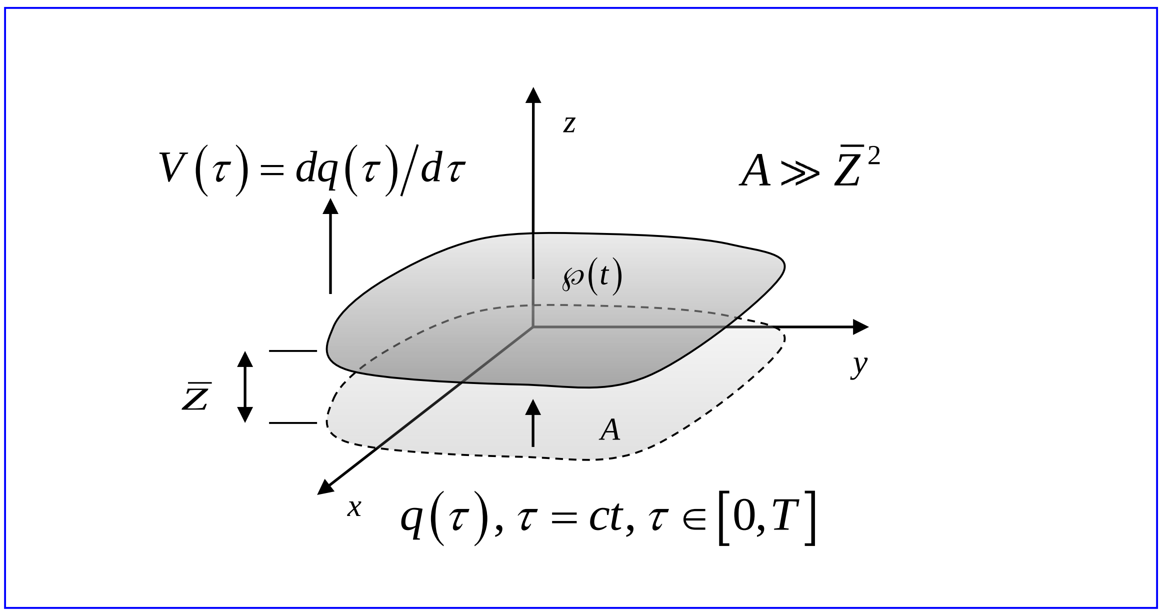
The blue-shaded boundary indicates the continuous motion of the front having a particular value of reflectance.



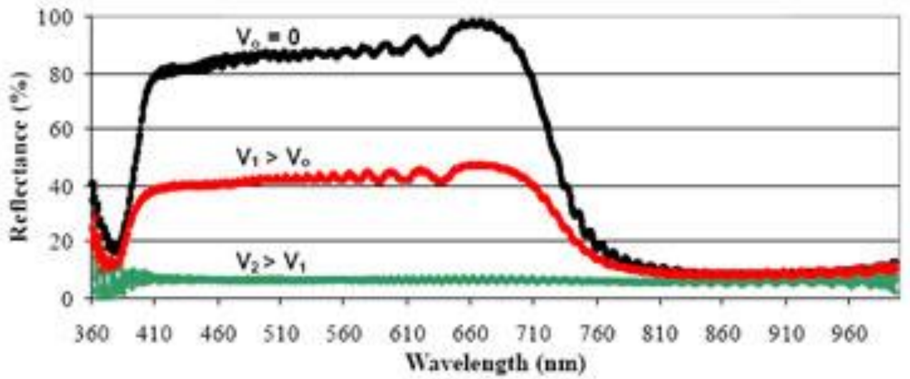
Paddle-Wheel Motion

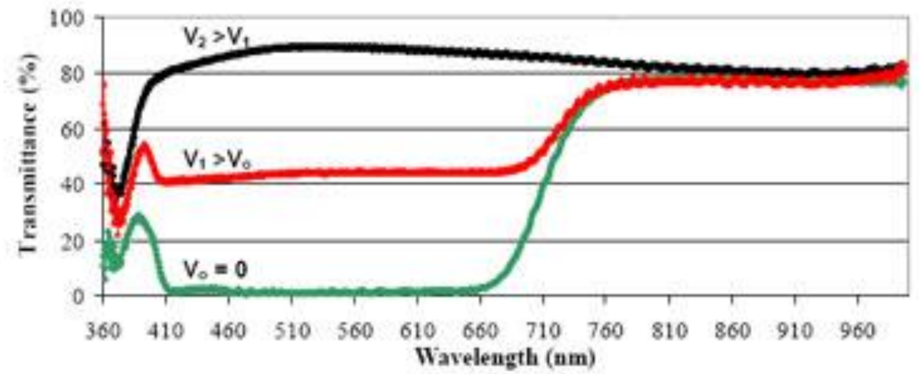
q(τ) monotonically increasing, q(0)=0





Transflection Characteristics*





Assume the reflectivity coefficient is given by:

$$R\left(k \in [k_l, k_u]\right) = \{0, \text{ or } 1$$

k is the wavenumber and $[k_l, k_u]$ is the region where switching is possible

*INVESTIGATION OF LIQUID CRYSTAL SWITCHABLE MIRROR OPTICAL CHARACTERISTICS FOR SOLAR ENERGY
P. Lemarchand; J.Doran; B.Norton
School of Physics, Dublin Energy Lab, Focas Institute, Dublin Institute of Technology, Dublin, Sep 15, 2017.

Dynamic Casimir forces due to reflective boundary conditions undergoing large motions - Formulation

- Use the Heisenberg picture: The initial state is fixed (at zero temp, in the vacuum state) and the operators evolve in time. The Heisenberg operator equations-of-motion are, in this case, Maxwell's equations for the field operators.

- Use the continuous Fock space representation with commutation relations:

$$\left[\hat{a}(\mathbf{k},s),\hat{a}^{\dagger}(\mathbf{k}',s')\right]=\delta^{3}(\mathbf{k}-\mathbf{k}')\,\delta_{ss'}$$

$$\left[\hat{a}(\mathbf{k},s),\hat{a}(\mathbf{k}',s')\right]=0$$

$$\left[\hat{a}^{\dagger}(\mathbf{k},s),\hat{a}^{\dagger}(\mathbf{k}',s')\right]=0$$

Dynamic Casimir forces due to reflective boundary conditions undergoing large motions - Formulation

- In the continuous Fock basis, at the starting time, the fields are given by:

$$\hat{\mathbf{E}}(\mathbf{r},t)=\frac{i}{(2\pi)^2}\sum_{s=1}^{2}\int\sqrt{\frac{hck}{2\varepsilon_0}}\left[\hat{a}(\mathbf{k},s)\boldsymbol{\varepsilon}(\mathbf{k},s)e^{i(\mathbf{k}\bullet\mathbf{r}-\omega t)}-h.c.\right]d^3k$$

$$\hat{\mathbf{B}}(\mathbf{r},t)=\frac{i}{(2\pi)^2}\sum_{s=1}^{2}\int\sqrt{\frac{h}{2ck\varepsilon_0}}\left[\hat{a}(\mathbf{k},s)\big(\mathbf{k}\times\boldsymbol{\varepsilon}(\mathbf{k},s)\big)e^{i(\mathbf{k}\bullet\mathbf{r}-\omega t)}-h.c.\right]d^3k$$

- Then determine the evolution of the operators:

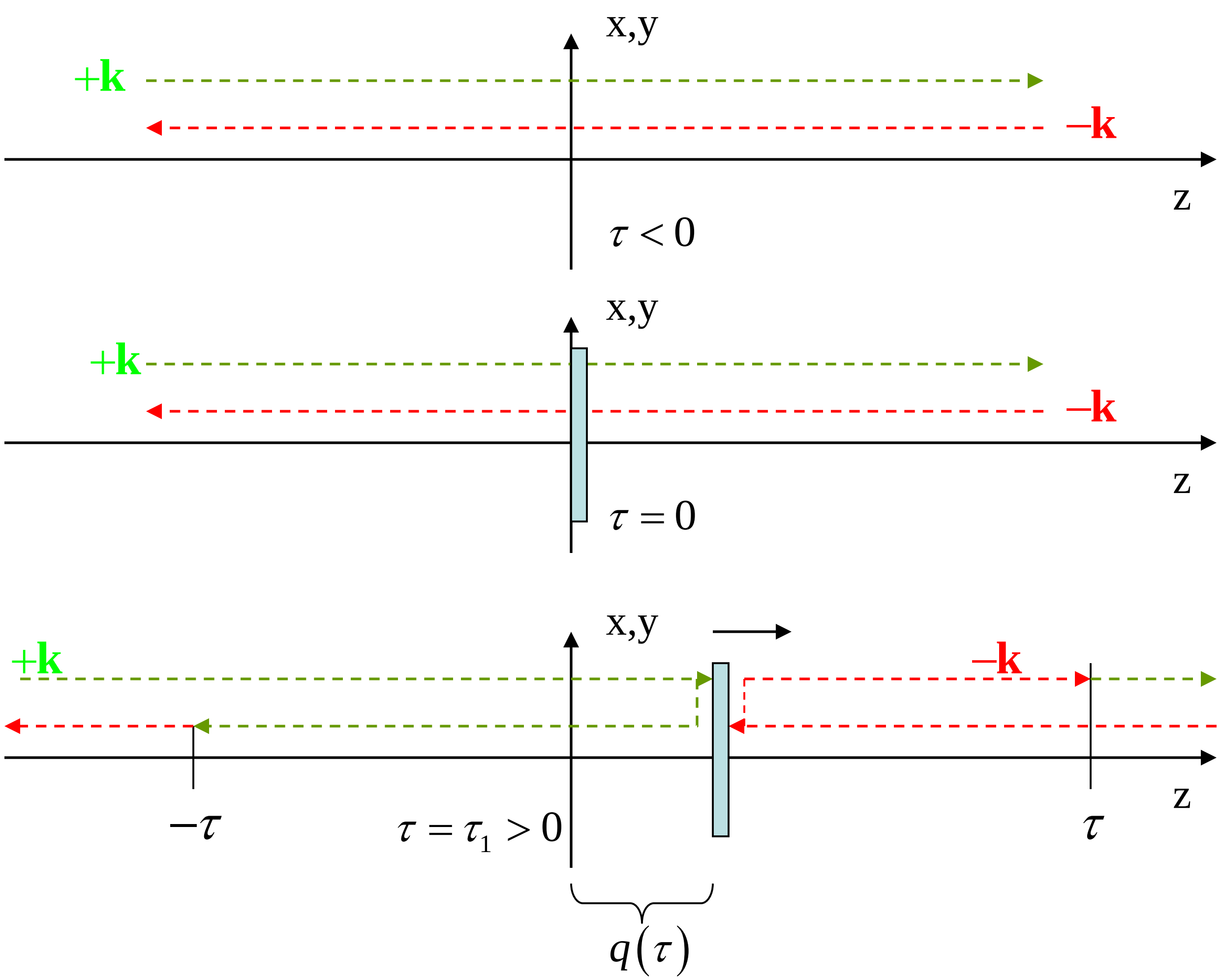
$$\hat{a}(\mathbf{k},s)\underbrace{\left(\sqrt{k},\text{ or }1/\sqrt{k}\right)\boldsymbol{\varepsilon}(\mathbf{k},s)\exp\big(i(\mathbf{k}\bullet\mathbf{x}-k\tau)\big)}_{\text{Mode function before "turn-on"}}$$

$$\hat{a}(\mathbf{k},s)=\text{annihilation operator}$$

$$\boldsymbol{\varepsilon}(\mathbf{k},s)=\text{polarization vector}$$

$$s=1,2:\text{polarization states}$$

The Situation Considered



Dynamic Casimir forces due to reflective boundary conditions undergoing large motions – Formulation

- Assume (1) the total amplitude of motion is much larger than a wavelength, (2) during one period the change in the surface velocity is $\ll$ c. Hence, the field operators evolve past $\tau = ct = 0$ according to:

$$\hat{\mathbf{E}}(\mathbf{r},\tau) = \frac{i}{(2\pi)^2}\sum_{s=1}^{2}\int\sqrt{\frac{hc\hat{k}_{\mathbf{k},s}}{2\varepsilon_0}}\left[\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},\tau)\hat{a}(\mathbf{k},s,\tau=0) - h.c.\right]d^3k$$

$$\hat{\mathbf{B}}(\mathbf{r},\tau) = \frac{1}{(2\pi)^2}\sum_{s=1}^{2}\int\sqrt{\frac{h}{2c\hat{k}_{\mathbf{k},s}\varepsilon_0}}\left[\left(\nabla\times\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},\tau)\hat{a}(\mathbf{k},s,\tau=0) + h.c.\right)\right]d^3k$$

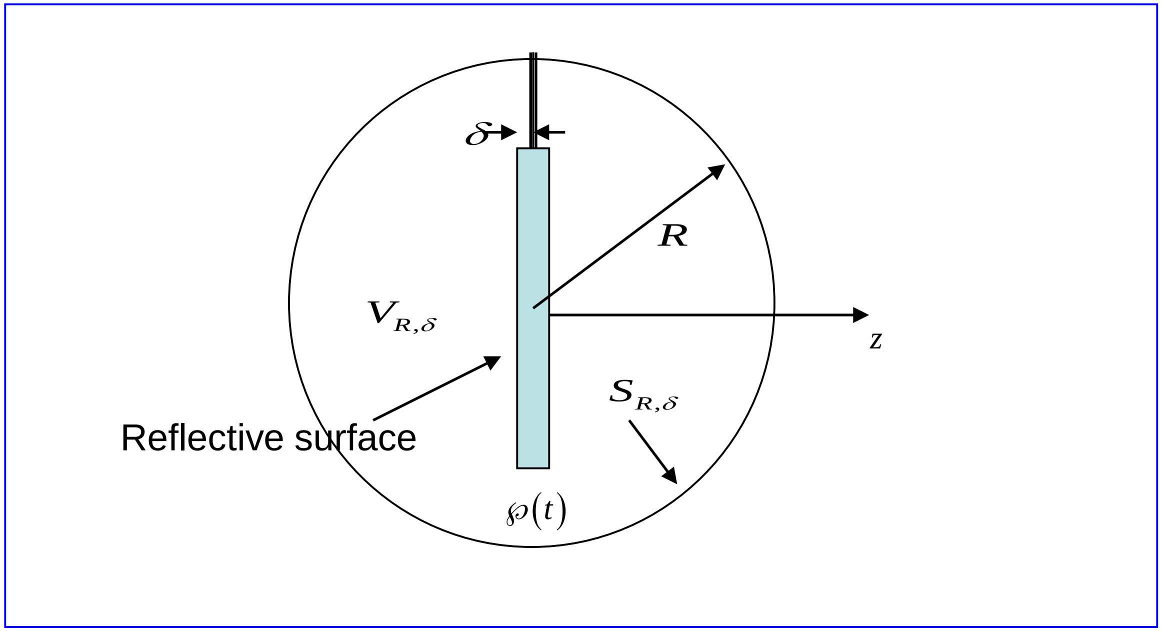
$\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},\tau)$ = transverse vector potential satisfying the wave equation and the boundary conditions of the electric, or magnetic field and the initial condition:

$$\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},\tau=0) = \boldsymbol{\varepsilon}(\mathbf{k},s)\left[e^{i(\mathbf{k}\bullet\mathbf{r}-k\tau)}\right]_{\tau=0}$$

- $\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},\tau)$ is an analytic signal. $\therefore$ the frequency, $kc,$ is positive.
- $c\hat{k}_{\mathbf{k},s}(\mathbf{r},\tau)$ represents the slowly varying evolution of the frequency (derivative of the eikonal) at $(\mathbf{r},\tau>0)$.

Dynamic Casimir forces due to reflective boundary conditions undergoing large motions - Formulation

- Integrate the Lorentz force operator per unit volume over the volume (below), apply the divergence theorem and let $R \to \infty$ and $\delta \to 0$



- Then the force on the field, is ....

Dynamic Casimir forces due to reflective boundary conditions undergoing large motions - Formulation

$$\hat{\mathbf{F}}_F\left(t\right)=\varepsilon_0\,\frac{d}{d\tau}\int_0^{\tau}d\tilde{\tau}\int_{V_{R,\delta}}\underline{\hat{\boldsymbol{\sigma}}}\bullet\mathbf{n}_{V_{R,\delta}}\;dS_{R,\delta}-\frac{1}{c}\frac{d}{d\tau}\int_{V_{R,\delta}}\hat{\mathbf{S}}\,dr^3$$

$$\left(\underline{\hat{\boldsymbol{\sigma}}}\right)_{ij}=\text{Maxwell stress operator}$$

$$=\varepsilon_0\left[\left(\hat{E}_i\hat{E}_j-\tfrac{1}{2}\delta_{ij}\hat{E}^2\right)+c^2\left(\hat{B}_i\hat{B}_j-\tfrac{1}{2}\delta_{ij}\hat{B}^2\right)\right]$$

$$\hat{\mathbf{S}}=\text{Symmetrized Poynting vector operator}$$

$$=\frac{1}{2\mu_0}\left[\hat{\mathbf{E}}\left(\mathbf{r},\tau\right)\times\hat{\mathbf{B}}\left(\mathbf{r},\tau\right)-\hat{\mathbf{B}}\left(\mathbf{r},\tau\right)\times\hat{\mathbf{E}}\left(\mathbf{r},\tau\right)\right]$$

$\mathbf{n}_{V_{R,\delta}}$ = unit normal to both surfaces of the reflective lamina

Averaging over the initial state - Example

$$\varepsilon_0\mu_0\frac{d}{d\tau}\int\langle vac|\hat{\mathbf{S}}|vac\rangle d^3r =$$

$$-\frac{d}{d\tau}\frac{ihc}{4(2\pi)^3}\int\sum_{s=1}^{2}\sum_{s'=1}^{2}\int d^3k\int d^3k'\sqrt{\frac{\hat{k}_{\mathbf{k},s}}{\hat{k}_{\mathbf{k}',s'}}}\langle vac|\begin{bmatrix}\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},\tau)\times(\nabla\times\boldsymbol{\Phi}_{\mathbf{k}',s'}(\mathbf{r},\tau))\hat{a}(\mathbf{k},s)\hat{a}(\mathbf{k}',s') \\ +\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},\tau)\times(\nabla\times\boldsymbol{\Phi}^{\bullet}{}_{\mathbf{k}',s'}(\mathbf{r},\tau))\hat{a}(\mathbf{k},s)\hat{a}^\dagger(\mathbf{k}',s') \\ -\boldsymbol{\Phi}^*_{\mathbf{k},s}(\mathbf{r},\tau)\times\nabla\times\boldsymbol{\Phi}_{\mathbf{k}',s'}(\mathbf{r},\tau)\hat{a}^\dagger(\mathbf{k},s)\hat{a}(\mathbf{k}',s') \\ -\boldsymbol{\Phi}^*_{\mathbf{k},s}(\mathbf{r},\tau)\times\nabla\times\boldsymbol{\Phi}^{\bullet}{}_{\mathbf{k}',s'}(\mathbf{r},\tau)\hat{a}^\dagger(\mathbf{k},s)\hat{a}^\dagger(\mathbf{k}',s') \\ -\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},\tau)\times(\nabla\times\boldsymbol{\Phi}_{\mathbf{k}',s'}(\mathbf{r},\tau))\hat{a}(\mathbf{k}',s')\hat{a}(\mathbf{k},s) \\ +\boldsymbol{\Phi}^*_{\mathbf{k},s}(\mathbf{r},\tau)\times(\nabla\times\boldsymbol{\Phi}_{\mathbf{k}',s'}(\mathbf{r},\tau))\hat{a}(\mathbf{k}',s')\hat{a}^\dagger(\mathbf{k},s) \\ -\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},\tau)\times(\nabla\times\boldsymbol{\Phi}^{\bullet}{}_{\mathbf{k}',s'}(\mathbf{r},\tau))\hat{a}^\dagger(\mathbf{k}',s')\hat{a}(\mathbf{k},s) \\ +\boldsymbol{\Phi}^*_{\mathbf{k},s}(\mathbf{r},\tau)\times(\nabla\times\boldsymbol{\Phi}^{\bullet}{}_{\mathbf{k}',s'}(\mathbf{r},\tau))\hat{a}^\dagger(\mathbf{k}',s')\hat{a}^\dagger(\mathbf{k},s)\end{bmatrix}|vac\rangle d^3r$$

$$\hat{a}(k)|vac\rangle=\langle vac|\hat{a}^\dagger(k)=0\quad\Downarrow$$

$$\varepsilon_0\mu_0\frac{d}{d\tau}\int\langle vac|\hat{\mathbf{S}}|vac\rangle d^3r$$

$$=\frac{d}{d\tau}\frac{ihc}{4(2\pi)^3}\int\sum_{s=1}^{2}\sum_{s'=1}^{2}\int d^3k\int d^3k'\sqrt{\frac{\hat{k}_{\mathbf{k},s}}{\hat{k}_{\mathbf{k}',s'}}}\begin{bmatrix}\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},\tau)\times(\nabla\times\boldsymbol{\Phi}^{\bullet}{}_{\mathbf{k}',s'}(\mathbf{r},\tau)) \\ -\boldsymbol{\Phi}^*_{\mathbf{k}',s'}(\mathbf{r},\tau)\times(\nabla\times\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},\tau))\end{bmatrix}\langle vac|\hat{a}(\mathbf{k},s)\hat{a}^\dagger(\mathbf{k}',s')|vac\rangle d^3r$$

$$\left[\hat{a}(\mathbf{k},s),\hat{a}^\dagger(\mathbf{k}',s')\right]=\delta^3(\mathbf{k}-\mathbf{k}')\delta_{s\,s'}\quad\Downarrow$$

$$\varepsilon_0\mu_0\frac{d}{d\tau}\int\langle vac|\hat{\mathbf{S}}|vac\rangle d^3r=\frac{d}{d\tau}\frac{hc}{2(2\pi)^3}\sum_{s=1}^{2}\int d^3r\int d^3k\,\mathrm{Im}\left[\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},\tau)\times(\nabla\times\boldsymbol{\Phi}^{\bullet}{}_{\mathbf{k},s}(\mathbf{r},\tau))\right]$$

Dynamic Casimir forces due to reflective boundary conditions undergoing large motions - Formulation

- By symmetry, there is only a z-component of force.
- Since $\bar{Z} \ll \sqrt{A} \rightarrow$ No x or y dependence of $\langle F_z \rangle$
- Take the quantum average (w.r.t. the initial state) and obtain the average force on the reflective surface, $\langle F_z \rangle$:

$$\langle F_z \rangle = -\langle \hat{\mathbf{z}} \bullet \hat{\mathbf{F}}_F \rangle$$

$$= -\varepsilon_0 \frac{d}{d\tau} \int_0^\tau d\tilde{\tau} \int_{\wp(t)} \langle vac | \hat{\mathbf{z}} \bullet \hat{\underline{\boldsymbol{\sigma}}} \bullet \mathbf{n}_{\wp(\tau)} | vac \rangle \, dS_\wp + A \frac{1}{c} \frac{d}{d\tau} \int \langle vac | \hat{\mathbf{z}} \bullet \hat{\mathbf{S}} | vac \rangle \, d^3 r$$

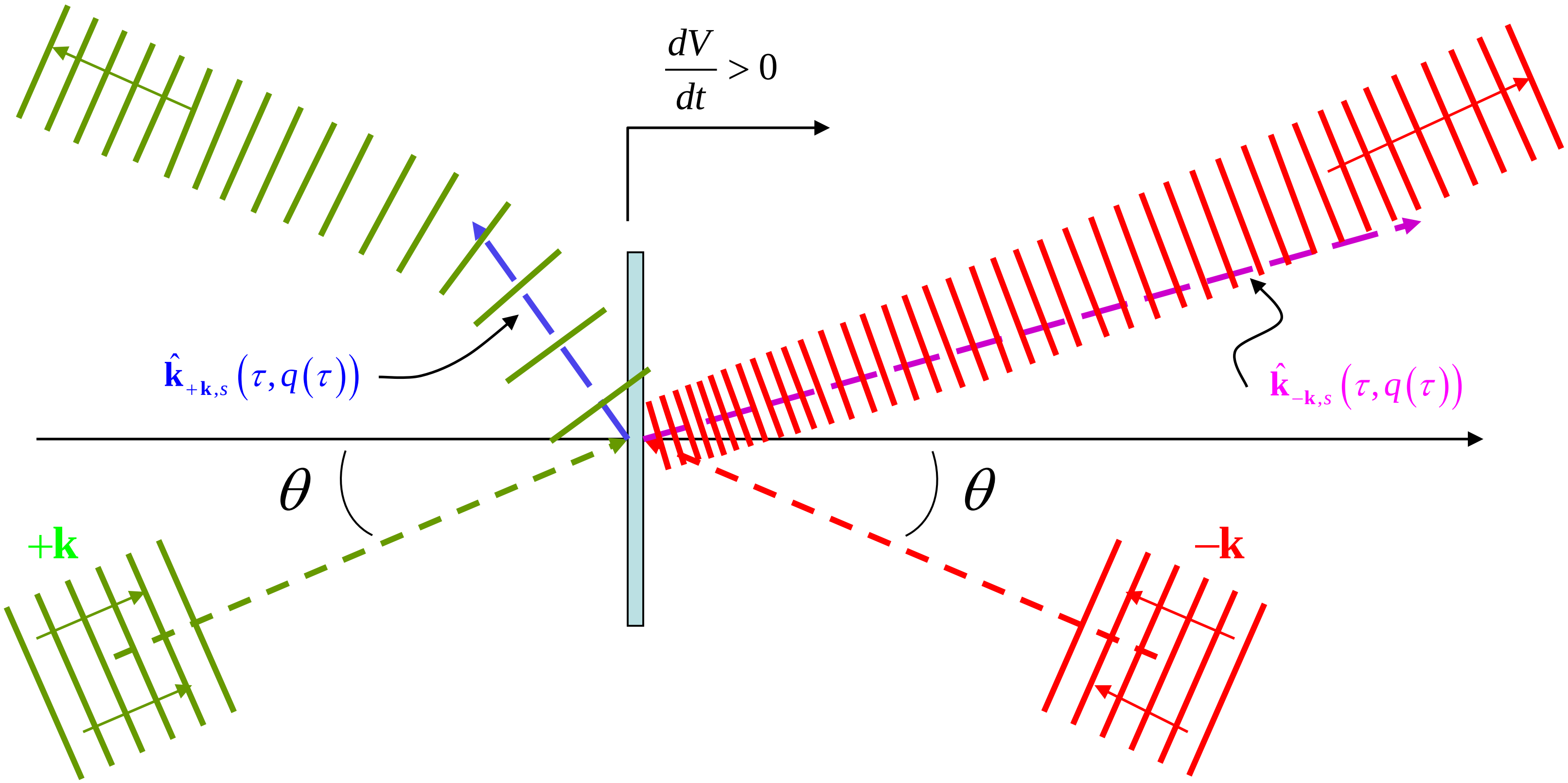
- Evaluate the average force in terms of $\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)$:

$$\langle F_z \rangle = \frac{hc}{2(2\pi)^3} \hat{\mathbf{z}} \bullet \frac{d}{d\tau} \sum_{s=1}^{2} \int d^3 r \int d^3 k \, R(k) \, \mathrm{Im}\left[\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},\tau) \times \left(\nabla \times \boldsymbol{\Phi}^{\bullet}_{\mathbf{k},s}(\mathbf{r},\tau) \right) \right]$$

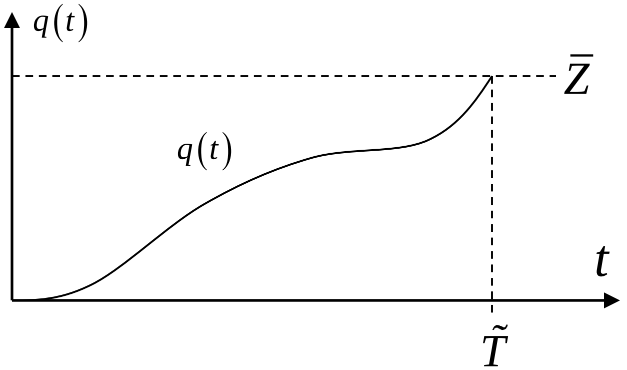
$$-\tfrac{1}{2} A \frac{hc}{2(2\pi)^4} \frac{d}{d\tau} \int_0^\tau d\tilde{\tau} \sum_{S=1,-1} S \sum_{s=1}^{2} \int R(k) \, d^3 k \left[\begin{array}{l} \hat{k} \left\{ \begin{array}{l} \left| \hat{\mathbf{x}} \bullet \boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},\tilde{\tau}) \right|^2 + \left| \hat{\mathbf{y}} \bullet \boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},\tilde{\tau}) \right|^2 \\ - \left| \hat{\mathbf{z}} \bullet \boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},\tilde{\tau}) \right|^2 \end{array} \right\}_{z=q+S\zeta} \\ + \dfrac{1}{\hat{k}} \left\{ \begin{array}{l} \left| \hat{\mathbf{x}} \bullet \left(\nabla \times \boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},\tilde{\tau}) \right) \right|^2 + \left| \hat{\mathbf{y}} \bullet \left(\nabla \times \boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},\tilde{\tau}) \right) \right|^2 \\ - \left| \hat{\mathbf{z}} \bullet \left(\nabla \times \boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},\tilde{\tau}) \right) \right|^2 \end{array} \right\}_{z=q+S\zeta} \end{array} \right]$$

- All terms have the factor $\int R(k) k^3 dk, \; k = |\mathbf{k}|$

General Character of the Mode Functions



Average Casimir Force: Dimensional Analysis



- Suppose $\exists$ a motion, bounded by $\bar{Z}$ and $\tilde{T} = T/c$, that produces a $\langle F_z \rangle \neq 0$.

  What is $\langle f_z \rangle_{\tilde{T}} = \frac{1}{\tilde{T}} \int_0^{\tilde{T}} dt \langle F_z(t)/A \rangle$ - aside from a dependence upon $\chi(t) = q(t/\tilde{T}) / \bar{Z}$?

- Without h there is no Casimir effect. Also h is the only factor that has units of mass

$$\begin{array}{ccccc} \langle f_z \rangle_{\tilde{T}} & = & h & \times & (?) \\ \frac{kg-m}{s^2-m^2} & & \frac{kg-m^2}{s} & & \frac{1}{m^3-s} \end{array}$$

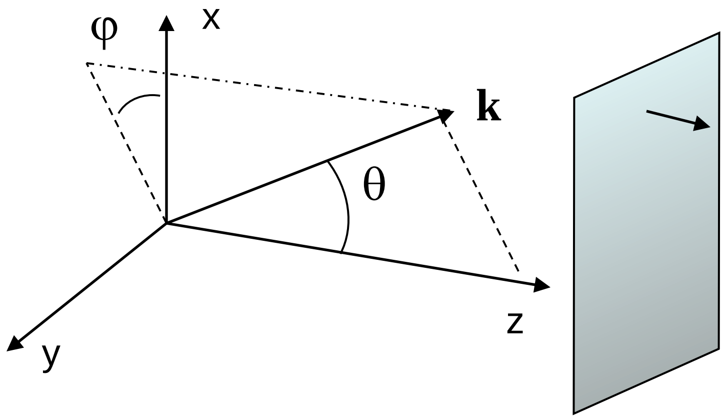
- Every term of $F_z(t)/A$ has the factor $\int_{k_l}^{k_u} dk\, R(k) k^3, k =$ waveno.of initial $|vac\rangle$ mode

$$\begin{array}{ccccccc} \langle f_z \rangle_{\tilde{T}} & = & h & \times & \left(\int_{k_l}^{k_u} dk\, R(k) k^3 \right) & \times & (?) \\ \frac{kg-m}{s^2-m^2} & & \frac{kg-m^2}{s} & & \frac{1}{m^4} & & \frac{m}{s} \end{array}$$

- The only velocity scale is $\bar{Z}/\tilde{T} = c\bar{Z}/T$, therefore:

$$\boxed{\langle f_z \rangle_{\tilde{T}} \approx hc \left(\int_{k_l}^{k_u} dk\, R(k) k^3 \right) \frac{\bar{Z}}{T} \times \left(\text{functional of } \chi(\tau/T), \tau \in [0, T) \right)}$$

A 1-D Approximation



- Define spherical coordinates in k-space as above. Then:

$$\int d^3k\, R(k)[\ldots] = \int_0^{2\pi} d\phi \int_0^{\pi/2} \sin\theta\, d\theta \int_0^{\bar{k}} dk R(k) k^2 [\ldots] = 2\pi \int_0^{\pi/2} \sin\theta\, d\theta \int_0^{\bar{k}} dk R(k) k^2 [\ldots]$$

- *Guess* that the $\langle F_z \rangle$ integrands have the main angular dependence $\cos(\theta)$
- Assume that the dominant wave vectors have $\theta << 1$

$$\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},\tau) = \Phi_{\mathbf{k}}(\mathbf{r},\tau)\boldsymbol{\varepsilon}(\mathbf{k},s)$$

$$\langle F_z \rangle / A \cong \frac{\hbar c}{2\pi} \frac{d}{d\tau} \int_0^\infty dk\, R(k) k^2 \int dz\; \mathrm{Im}\left[\Phi_{\mathbf{k}}(\mathbf{r},\tau) \nabla \Phi^*_{\mathbf{k}}(\mathbf{r},\tau)\right]$$

$$- \frac{\hbar c}{16(\pi)^2} \frac{d}{d\tau} \int_0^\tau d\tilde{\tau} \sum_{S=1,-1} S \int_0^\infty dk\, R(k) k^2 \left\{ \hat{k} \left|\Phi_{\mathbf{k}}(\mathbf{r},\tilde{\tau})\right|^2 + \frac{1}{\hat{k}} \left| \frac{\partial}{\partial z} \Phi_{\mathbf{k}}(\mathbf{r},\tilde{\tau}) \right|^2 \right\}_{z=q+S\zeta}$$

A 1-D Approximation – Solution of the scalar wave equation

$$\frac{\partial^2}{\partial z^2}\Phi_{\alpha k}(z,\tau)=\frac{\partial^2}{\partial \tau^2}\Phi_{\alpha k}(z,\tau)$$

$$\Phi_{\alpha k}(z,\tau=0)=e^{i\alpha kz}$$

$$\Phi_{\alpha k}(z=q(\tau),\tau)=0,\ \alpha=\pm 1$$

$$\Downarrow$$

$$\Phi_{+1,k}(z,\tau)=\begin{cases}\exp\left(ik(z-\tau)\right)-\exp\left(ikS_{+}(-z-\tau)\right), z\le q(\tau)\\ 0,\ \tau\ge z>q(\tau)\end{cases}$$

$$\Phi_{-1,k}(z,\tau)=\begin{cases}\exp\left(ik(-z-\tau)\right)-\exp\left(ikS_{-}(z-\tau)\right), z\ge q(\tau)\\ 0,-\tau\le z<q(\tau)\end{cases}$$

$$S_{+}\left(-q(\tau)-\tau\right)=q(\tau)-\tau,\quad S_{-}\left(q(\tau)-\tau\right)=-q(\tau)-\tau$$

- Note that since $V(\tau)<1$ the following sequences are convergent:

$$S_{\pm}(\xi)=\xi\pm 2\lim_{k\to\infty}q\left(\xi_{k,\pm}\right)$$

$$\xi_{k,\pm}=-\xi\mp q\left(\xi_{k-1}\right),\ k\ge 1$$

$$\xi_0=-\xi$$

A 1-D Approximation

- $\mathbf{\Phi}_{\mathbf{k},s}(\mathbf{r},t)$represents the evolution of the vector field operator from the initial plane wave configuration in the vacuum state having wave vector k.
- For each half space, there is an incident wave with wave vector and a reflected wave, also planar.
- Assume that (1) the total amplitude of motion is much larger than a wavelength, (2) during the time required for the passage of one wavelength, the relative change in the surface velocity is very small.

- As an example of one consequence of Assumption 1:

$$\sum_{\alpha=-1}^{+1}\int_{-\infty}^{\infty}\mathrm{Im}\left[\Phi_{\alpha k}\left(z,\tau\right)\frac{\partial}{\partial z}\Phi^{*}_{\alpha k}\left(z,\tau\right)\right]dz$$

$$= -k\int_{0}^{-q(\tau)-\tau} d\chi_{+}\left[1-\cos\left(k\left(\chi_{+}-2\tau-S_{+}\left(\chi_{+}\right)\right)\right)\right]\left[\frac{\partial S_{+}\left(\chi_{+}\right)}{\partial\chi_{+}}-1\right]$$

$$+k\int_{q(\tau)-\tau}^{0} d\chi_{-}\left[1-\cos\left(k\left(\chi_{-}+2\tau-S_{-}\left(\chi_{-}\right)\right)\right)\right]\left[\frac{\partial S_{-}\left(\chi_{-}\right)}{\partial\chi_{-}}-1\right]$$

- The cosine terms, resulting from the product of incident and reflected waves, have twice the freuency of the incident waves and can be neglected.
- Note: If $1/k \gg \bar{Z}$, the cosine terms dominate and in the limit the force becomes independent of k

A 1-D Approximation

- Neglecting all terms having incident $\times$ reflected products, we get the eikonal approximation:

$$\boxed{\begin{aligned}&\left\langle F_z(\tau)\right\rangle / A = \frac{\hbar c}{8\pi^2}\int_0^\infty dk\, R(k) k^3 \;\frac{d}{d\tau} q(\tau)\Lambda(\tau)\\ &\Lambda(\tau) = \frac{1}{q(\tau)}\left\{\int_{-\tau}^{q(\tau)} d\tilde{\tau}\,\frac{1}{1+V(\tau+\tilde{\tau})} - \int_{q(\tau)}^{\tau} d\tilde{\tau}\,\frac{1}{1-V(\tau-\tilde{\tau})}\right\}\end{aligned}}$$

- Obviously, if $V = dq/d\tau$ = constant, the force vanishes
- $\Lambda(\tau)$ depends only on the past history of $V \rightarrow$ system is causal
- $\left\langle f_z \right\rangle_{\tilde{T}} = \dfrac{1}{\tilde{T}A}\left\langle F_z(\tau)\right\rangle = \dfrac{1}{16\pi^3} hc\left(\int_{k_l}^{k_u} dk\, R(k) k^3\right)\dfrac{\bar{Z}}{T} \times \Lambda(\tau = T)$

  (in agreement with the dimensional analysis)

A Class of Accelerating Motions

- Consider surface motions that are powers of time:

$$q(\tau) = \bar{Z}(\tau/T)^N , \tau \in [0,T]$$

- The velocity may be written:

$$V(\tau) = \bar{V}(T)^{N-1}, \bar{V} = N\bar{Z}/T$$

- so that $\bar{V}$ is the maximum speed during the maneuver.

- $\Lambda(\tau = T)$ epitomizes the asymmetry of the field due to the surface motion. This is given by:

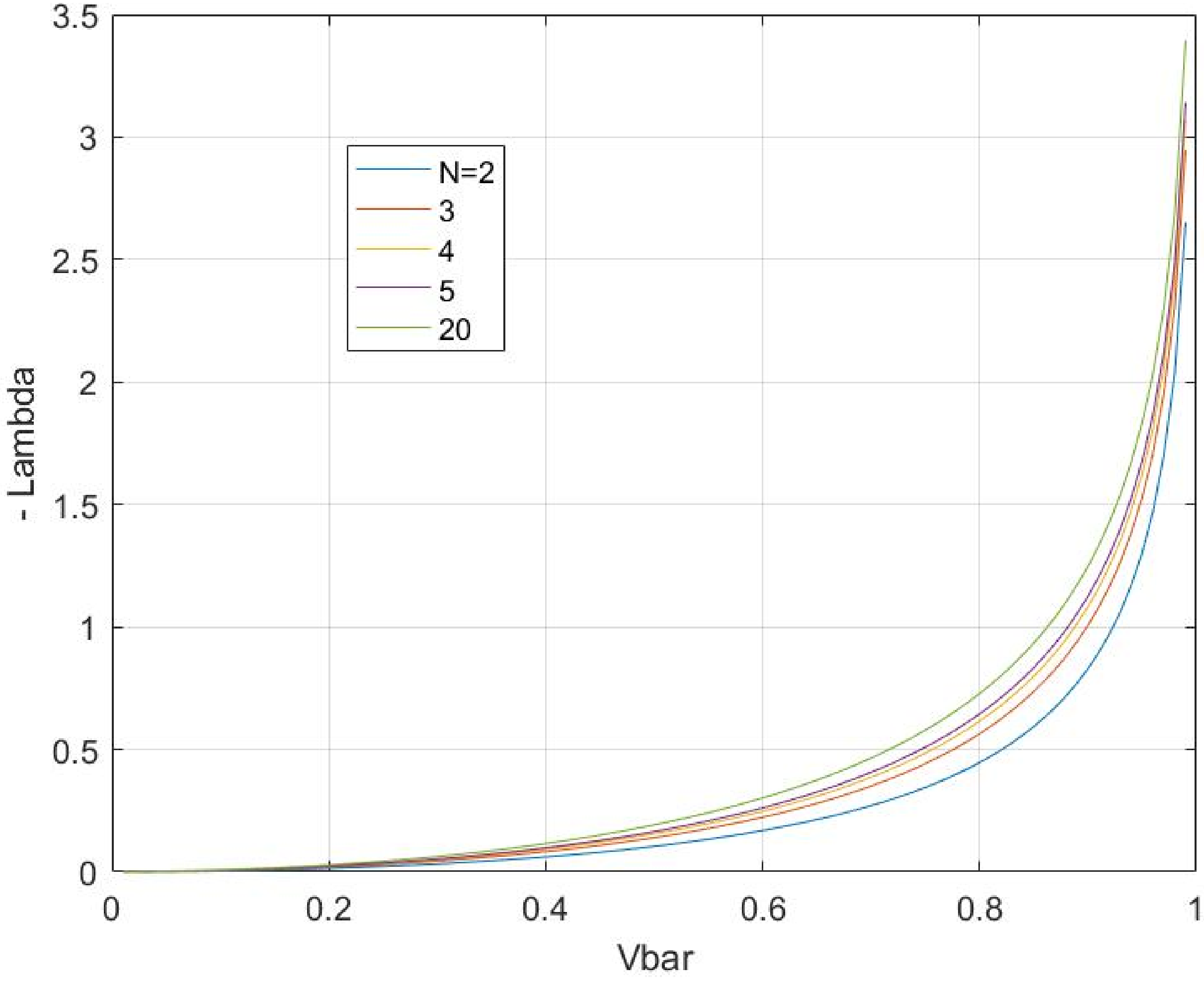
$$\Lambda(\tau = T) = \sum_{m=0}^{\infty} \frac{\bar{V}^m}{m(N-1)+1} \left[(-1)^m \left(\frac{N}{\bar{\bar{V}}} + 1 \right) - \left(\frac{N}{\bar{\bar{V}}} - 1 \right) \right]$$

- The change in the normalized velocity in the time reqiured by a wave with wave number k to travel one wavelength is:

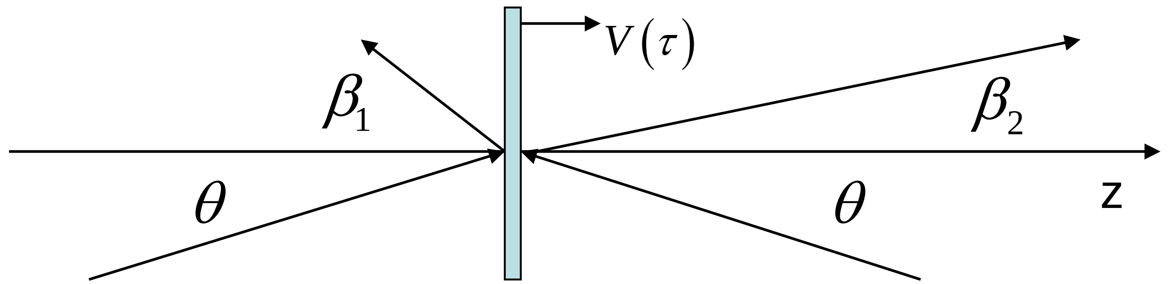
$$\frac{\Delta V}{\bar{\bar{V}}} = (N-1)) \frac{\lambda}{T} \left(\frac{\tau}{T} \right)^{N-2} \approx O\left(\frac{\lambda}{c\tilde{T}} \right)$$

A Class of Accelerating Motions

$$\Lambda\left(\tau=T\right)=\sum_{m=0}^{\infty}\frac{\bar{V}^{m}}{m\left(N-1\right)+1}\left[\left(-1\right)^{m}\left(\frac{N}{\bar{\bar{V}}}+1\right)-\left(\frac{N}{\bar{\bar{V}}}-1\right)\right]$$



The 3-D Problem – The Eikonal Approximation



- Mixed incident $\times$ reflected terms neglected
- $\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t) = \exp\Big(i\big(\mathbf{k}(\tau)\bullet\mathbf{r} - k\tau\big)\Big)\boldsymbol{\varepsilon}\big(\mathbf{k}(\tau),s\big)$

$$\mathrm{Im}\Big[\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\times\big(\nabla\times\boldsymbol{\Phi}^{\bullet}{}_{\mathbf{k},s}(\mathbf{r},t)\big)\Big] = k\kappa(\mathbf{r},\theta,\tau)$$

$$\hat{k}\left\{\left|\mathbf{u}\bullet\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\right|^2\right\} = \frac{1}{\hat{k}}\left|\mathbf{u}\bullet\big(\nabla\times\boldsymbol{\Phi}_{\mathbf{k},s}(\mathbf{r},t)\big)\right|^2 = k\left|\big(\mathbf{u}-(\hat{\mathbf{x}}\bullet\mathbf{u})\mathbf{u}\big)\bullet\kappa(\mathbf{r},\theta,\tau)\theta\right|$$

$$\kappa(\mathbf{r},\theta,\tau) = \mathbf{k}(\mathbf{r},\theta,\tau)/k$$

- Assumption 2 $\rightarrow$ relativistic reflection conditions may be used:

$$\left|\kappa_1(\mathbf{r},\theta,\tilde{\tau})\right| = \frac{1-2V\cos\theta+V^2}{1-V^2},\ \cos\beta_1 = \frac{-2V+\left(1+V^2\right)\cos\theta}{1-2V\cos\theta+V^2}$$

$$\left|\kappa_2(\mathbf{r},\theta,\tilde{\tau})\right| = \frac{1+2V\cos\theta+V^2}{1-V^2},\ \cos\beta_2 = \frac{2V+\left(1+V^2\right)\cos\theta}{1+2V\cos\theta+V^2}$$

The 3-D Problem – The Eikonal Approximation

$$\left\langle F_z(\tau)\right\rangle / A = \frac{\hbar c}{(2\pi)^2}\left(\int_0^{\bar{k}} dk\, R(k) k^3\right)\frac{d}{d\tau}\left[q(\tau)\int_0^{\pi/2}\sin\theta\;\Gamma(\tau,\theta)\,d\theta\right]$$

$$\Gamma(\tau,\theta) = \frac{1}{q(\tau)}\left\{\int_{-\chi_1(\tau)}^{q(\tau)} d\tilde{\tau}\,\frac{\left(\cos\theta - V(\tau+\tilde{\tau})\right)}{1-V^2(\tau+\tilde{\tau})} - \int_{q(\tau)}^{\chi_2(\tau)} d\tilde{\tau}\,\frac{\left(\cos\theta + V(\tau-\tilde{\tau})\right)}{1-V^2(\tau-\tilde{\tau})}\right\}$$

$$\frac{d}{d\tau}\chi_1 = \frac{-2V + \left(1+V^2\right)\cos\theta}{1-2V\cos\theta+V^2},\quad \frac{d}{d\tau}\chi_2 = \frac{2V + \left(1+V^2\right)\cos\theta}{1+2V\cos\theta+V^2}$$

- When $V =$ constant, the force vanishes

The 3-D Problem – Lower and Upper Bounds

$V = 1 - v,\ v << 1:$

$$\left|\langle F_z \rangle / A\right| = \left|\langle \hat{F}_z \rangle / A\right| + O\left(v^2\right)$$

$$\left|\langle \hat{F}_z \rangle / A\right| \geq \frac{\hbar c}{8\pi^2}\left(\int_0^{\bar{k}} dk R(k) k^3\right)\left|\frac{d}{d\tau} q(\tau)\hat{\Lambda}(\tau)\right|$$

$$\hat{\Lambda}(\tau) = \frac{1}{q(\tau)}\left\{\int_{-\tau}^{q(\tau)} d\tilde{\tau}\,\frac{1}{1+V(\tau+\tilde{\tau})} - \int_{q(\tau)}^{\tau} d\tilde{\tau}\,\frac{1}{1-V(\tau-\tilde{\tau})}\right\}$$

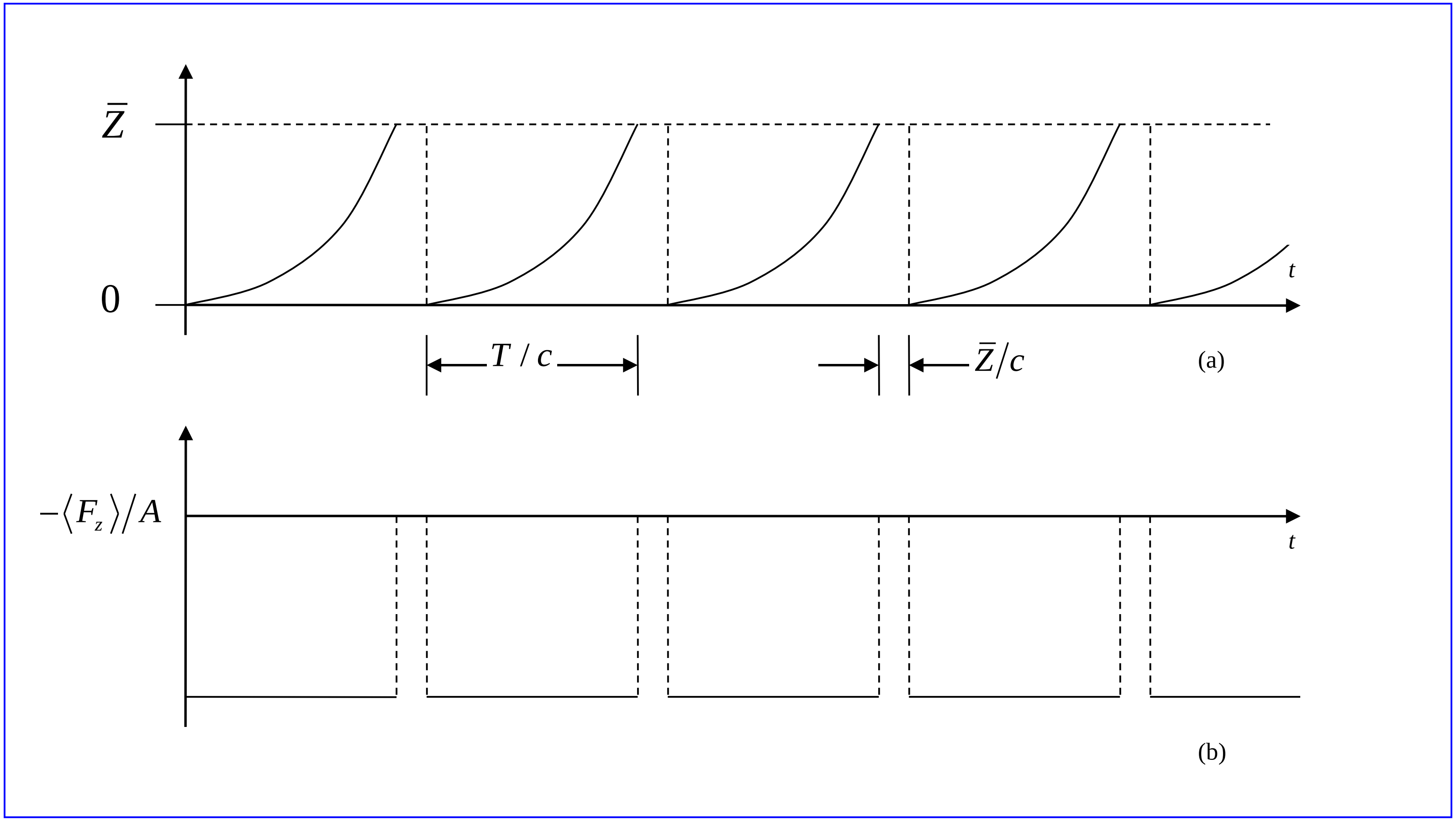
$V \ll 1:$

$$\left|\langle F_z \rangle / A\right| = \left|\langle \breve{F}_z \rangle / A\right| + O\left(V\right)$$

$$\left|\langle \breve{F}_z \rangle / A\right| \leq \frac{2\hbar c}{8\pi^2}\left(\int_0^{\bar{k}} dk R(k) k^3\right)\left|\frac{d}{d\tau} q(\tau)\breve{\Lambda}(\tau)\right|$$

$$\breve{\Lambda}(\tau) = \frac{1}{q(\tau)}\left\{\int_{-\tau}^{q(\tau)} d\tilde{\tau}\,\frac{1}{1+V(\tau+\tilde{\tau})} - \int_{q(\tau)}^{\tau} d\tilde{\tau}\,\frac{1}{1-V(\tau-\tilde{\tau})}\right\}$$

Average Force for the Case of Periodic Scans



(a) Cyclic waveform of the reflective surface position; (b) Force on the momentum exchange device.

Average Force for the Case of Periodic Scans

$$R(k) = \begin{cases} 1, & k \in [k_L, k_U] \\ 0, & otherwise \end{cases},$$

$$\left(\int_0^\infty dk\, R(k) k^3 \right) = \bar{\bar{k}}^3 \Delta \bar{k}$$

$$\bar{\bar{k}} = \left[\tfrac{1}{2}\left(k_U^2 + k_L^2\right) \cdot \tfrac{1}{2}\left(k_U + k_L\right) \right]^{1/3}$$

$$\Delta \bar{k} = k_U - k_L$$

$$\left\langle \langle F_z \rangle / A \right\rangle_t = \frac{\hbar c}{(4\pi)^2} \bar{\bar{k}}^3 \Delta \bar{k}\, \bar{\beta}\, \Lambda(T)$$

$$\Lambda(T) = \frac{1}{\bar{\bar{Z}}} \left\{ \int_{-T}^{\bar{Z}} d\tilde{\tau} \frac{1}{1 + V(\tau + \tilde{\tau})} - \int_{\bar{Z}}^{T} d\tilde{\tau} \frac{1}{1 - V(\tau - \tilde{\tau})} \right\}$$

$$\bar{\bar{k}} = \left[\tfrac{1}{2}\left(k_U^2 + k_L^2\right) \cdot \tfrac{1}{2}\left(k_U + k_L\right) \right]^{1/3}$$

$$\Delta \bar{k} = k_U - k_L$$

$$\bar{\beta} = 2\bar{Z} \big/ \left(T + \bar{Z}\right) \in [0,1)$$

Example: Periodic Motion with Power-Law Waveforms

- Plasma frquency $10^{14}\ Hz$ to $10^{16}\ Hz$, $\quad k_U = 2\times10^7\ (\lambda \cong 0.3\mu m)$

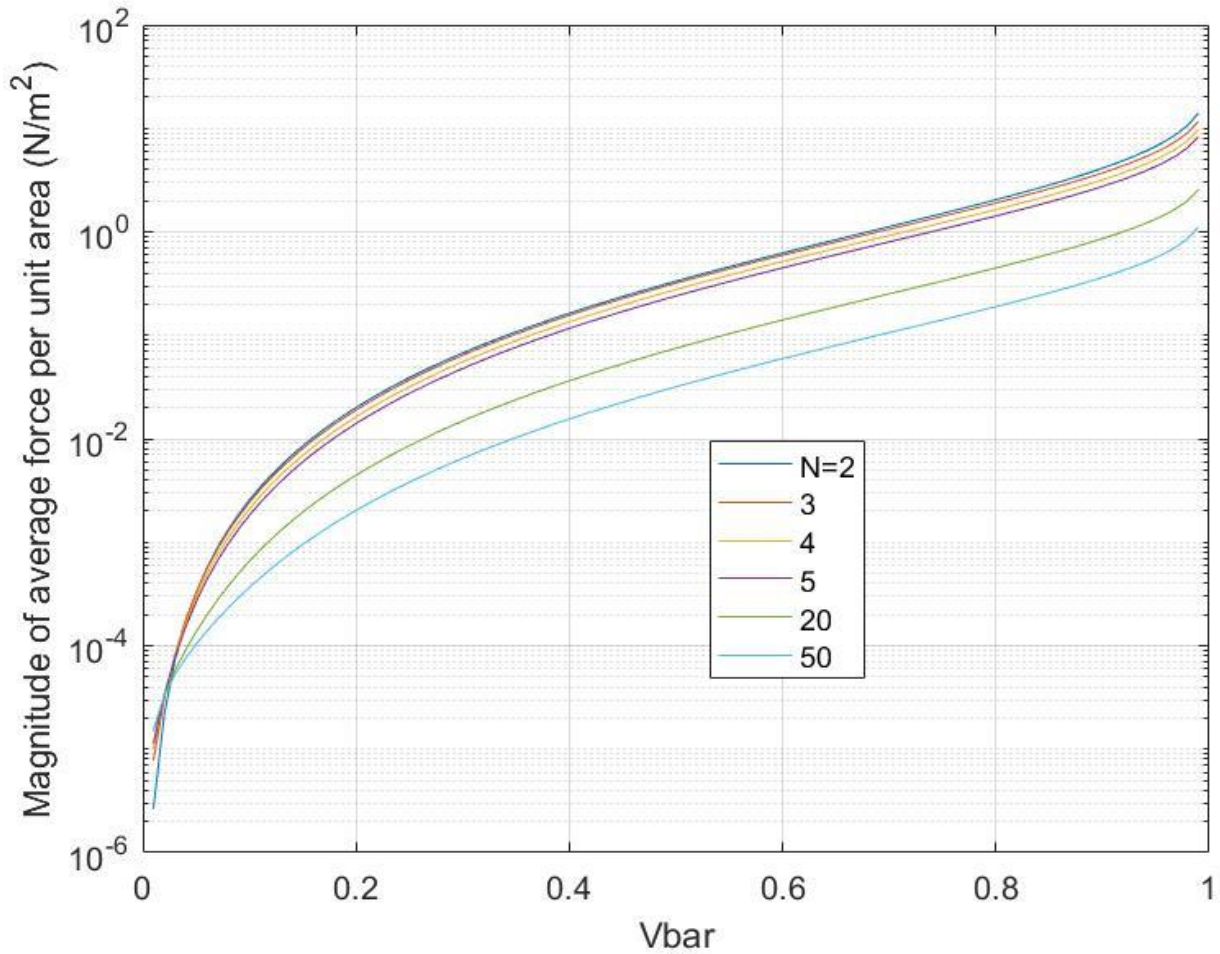


Figure 6. Force per unit area as a function of the maximum waveform velocity, integer powers

Example: Periodic Motion with Power-Law Waveforms

- Plasma frquency $10^{14}\ Hz$ to $10^{16}\ Hz$, $\quad k_U = 2\times10^{7}\ (\lambda \cong 0.3\mu m)$

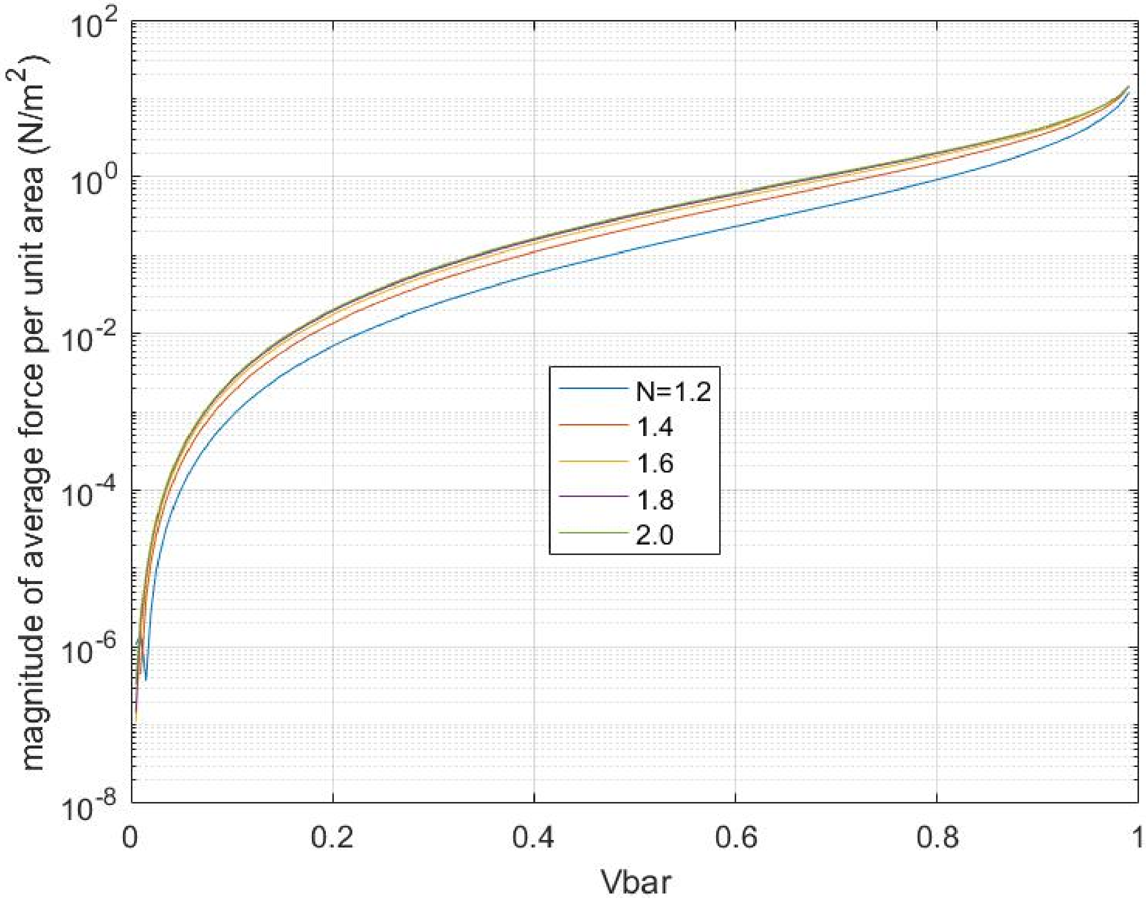
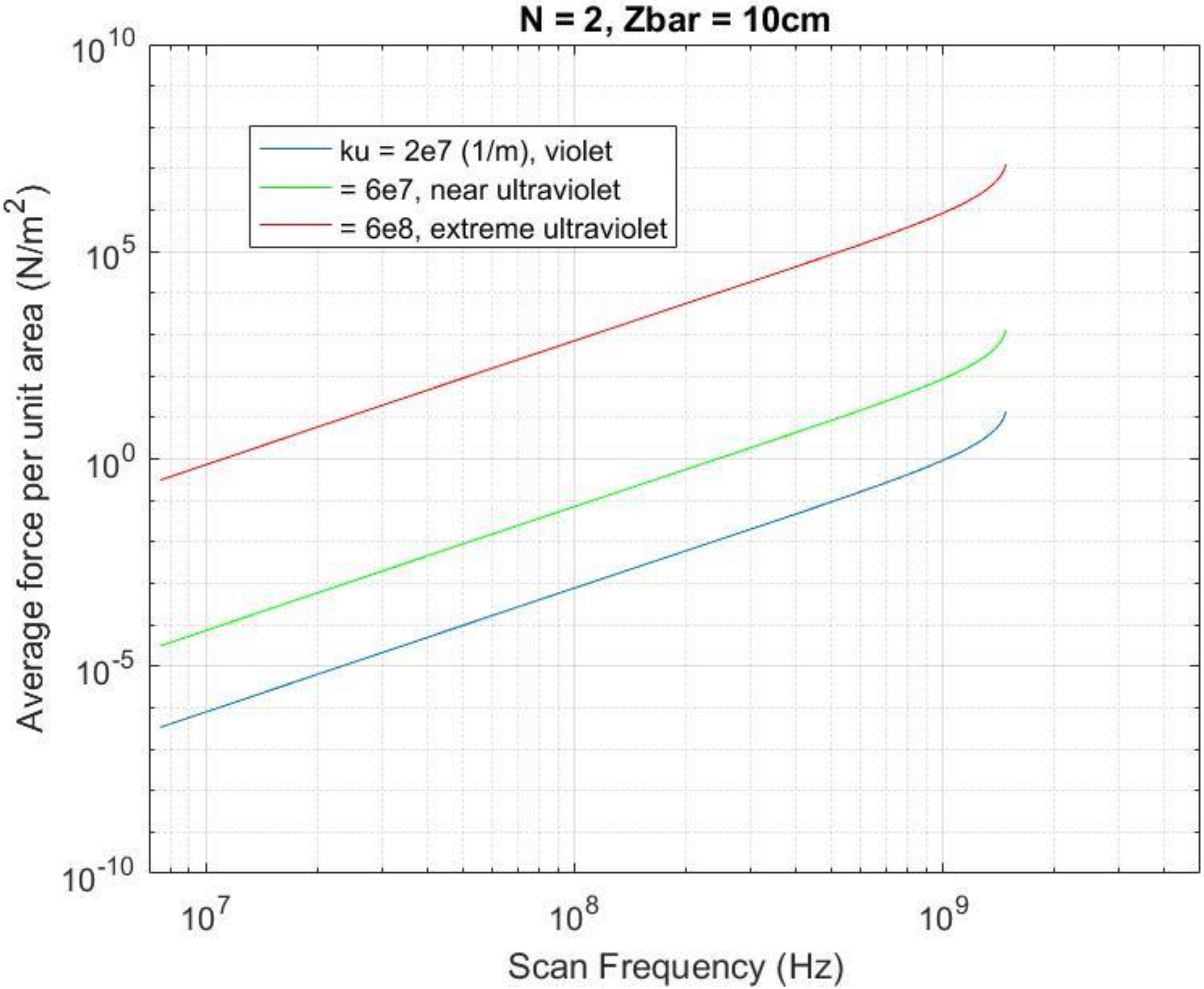
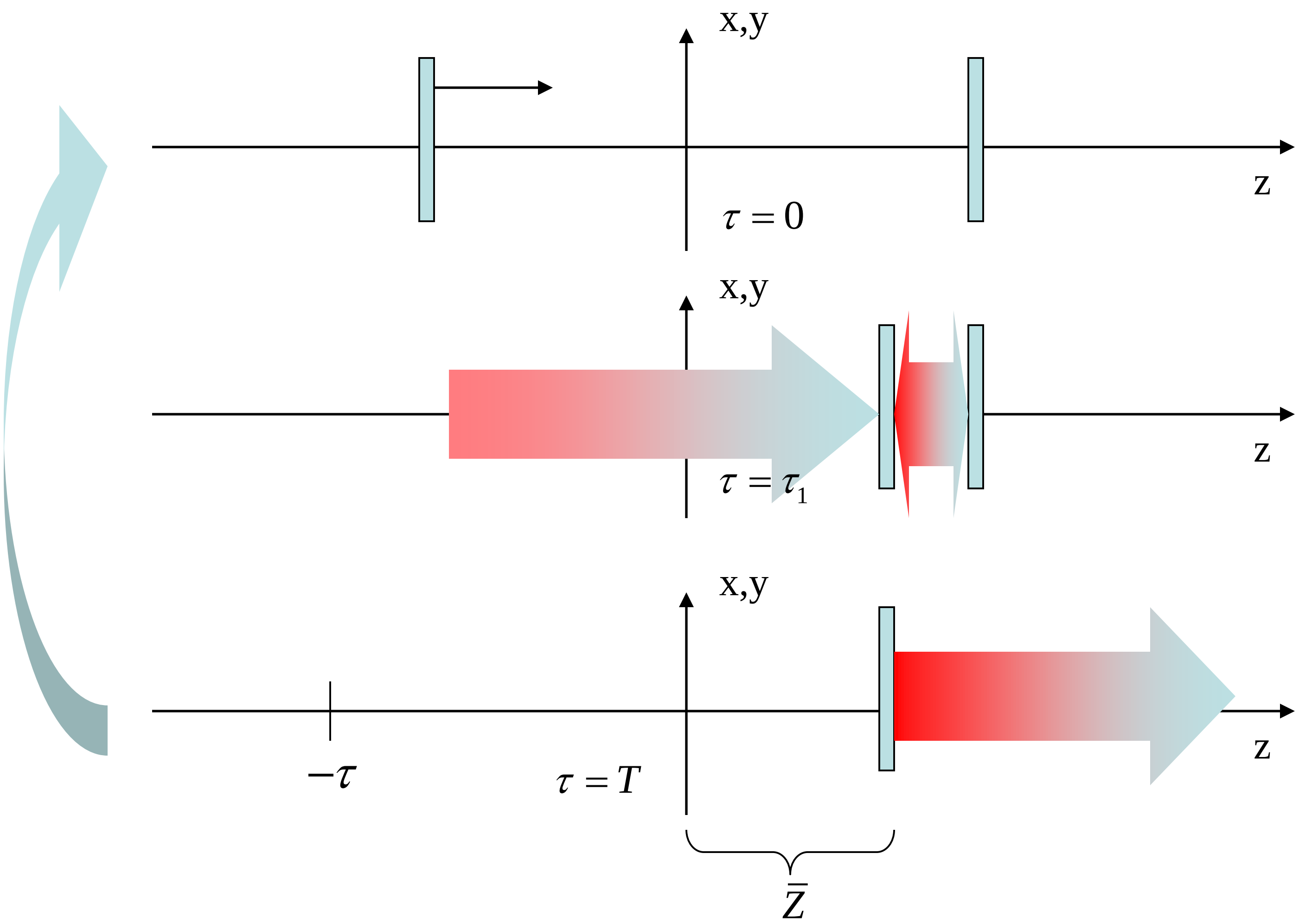


Figure 7. Force per unit area as a function of the maximum waveform velocity, fractional powers

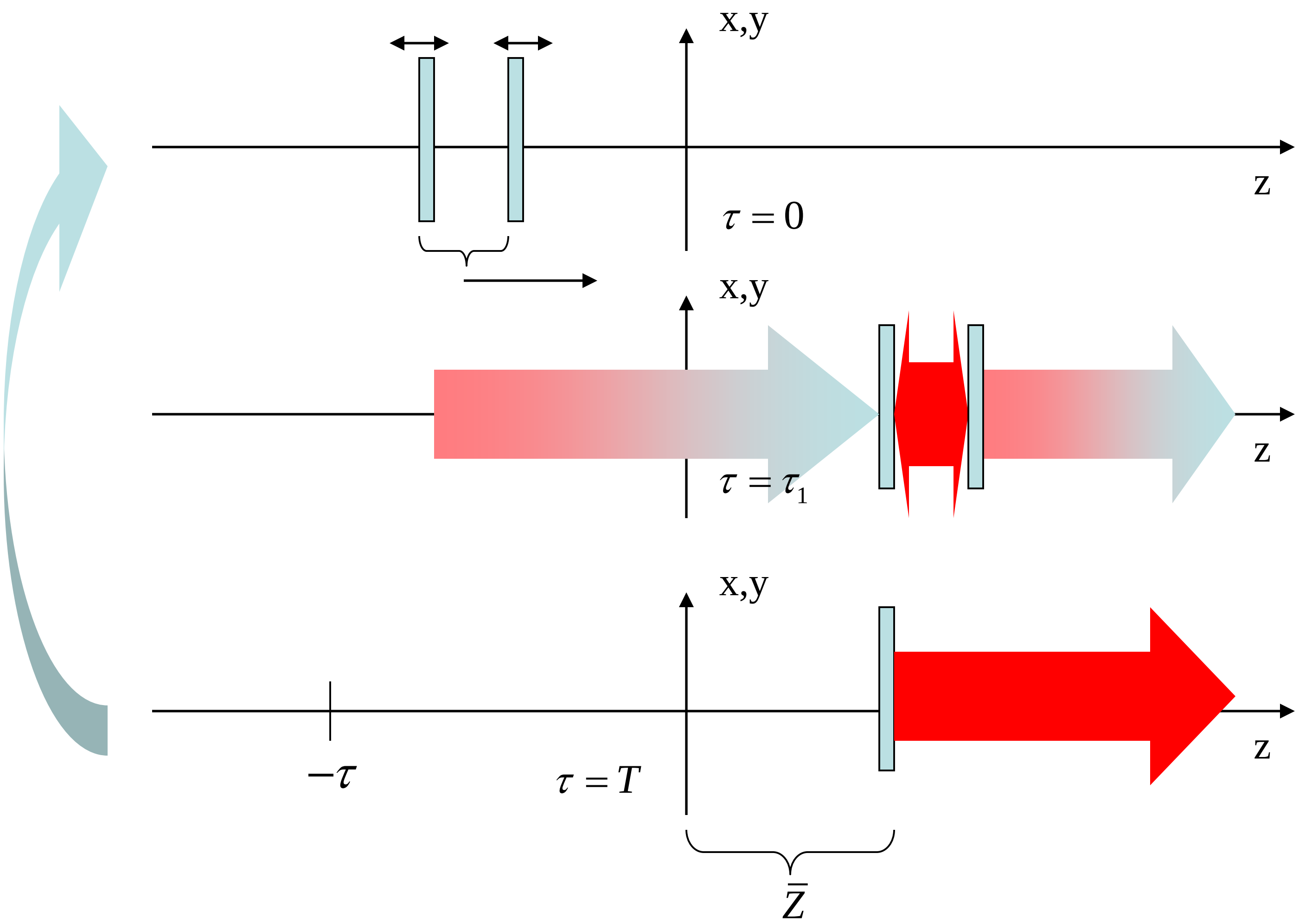


N = 2, Zbar = 10cm
ku = 2e7 (1/m), violet
= 6e7, near ultraviolet
= 6e8, extreme ultraviolet
Average force per unit area (N/m²)
Scan Frequency (Hz)

More Complex Operation: The Piston



More Complex Operation: Resonator



Concluding Remarks

- This paper re-examines the dynamic Casimir effect as a possible mechanism for propulsion and seeks large amplitude motion.
- An epitaxial stack of transparent/reflective laminae is proposed, wherein voltage switching creates large motion of a reflective surface without moving parts.
- Since previous analysis of the propulsive effect was restricted to motions much smaller than the wavelengths of importance, it is necessary to derive more general expressions.
- A class of accelerating, power-law, motions was examined and the forces computed.
- For motions of the reflective surface that are much larger than the wavelength range of significance, the approach taken here yields an eikonal approximation that may simplify calculations in more complicated cases.
- Restrictions:
  - Detailed dielectric function models not used – merely a wavelength range within which switching is possible
  - As for previous workers, the treatment is semi-quantum in that the epitaxial stack is modeled as a set of prescribed boundary conditions on the field operators.
  - **Use of two reflective surfaces (cavities) may enhance the effect by the finesse of the cavity**
- Despite these restrictions, if reasonable switching frequencies are possible, the propulsive forces may be quite significant.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