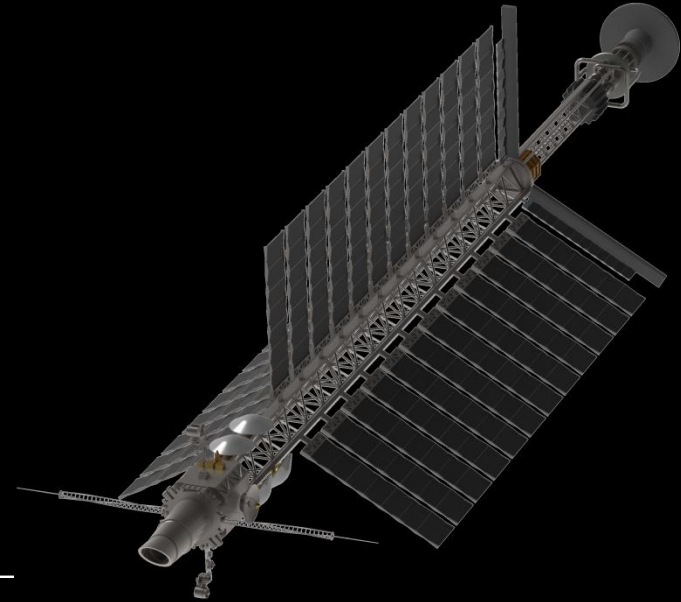


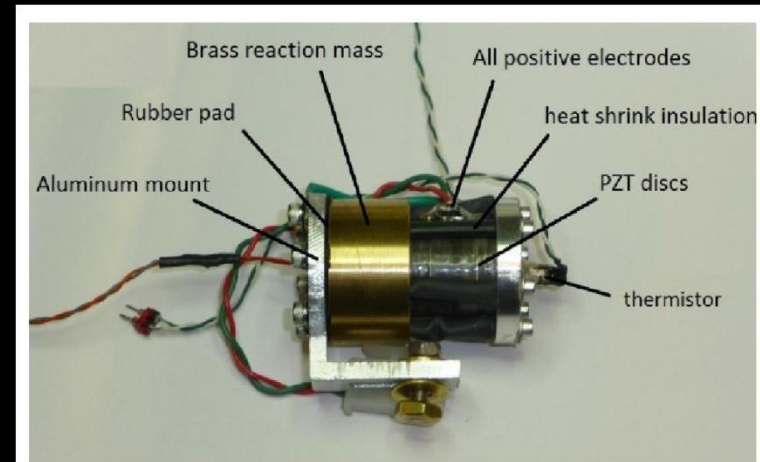
# Propellantless space propulsion from a gravitational effect sourced by energy fluctuations

José J. A. Rodal, Ph.D.

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1. 9 confusions in the literature
2. Correct formulation
3. Exact solution results
4. Conclusions



# Discrepancy between models and experiments



Tajmar, M., 2017, "Mach Effect Thruster Model," Acta Astronautica, 141, pp. 8-16

the large discrepancy between

theory and experimental results persists after some 27 years of development and thus raises

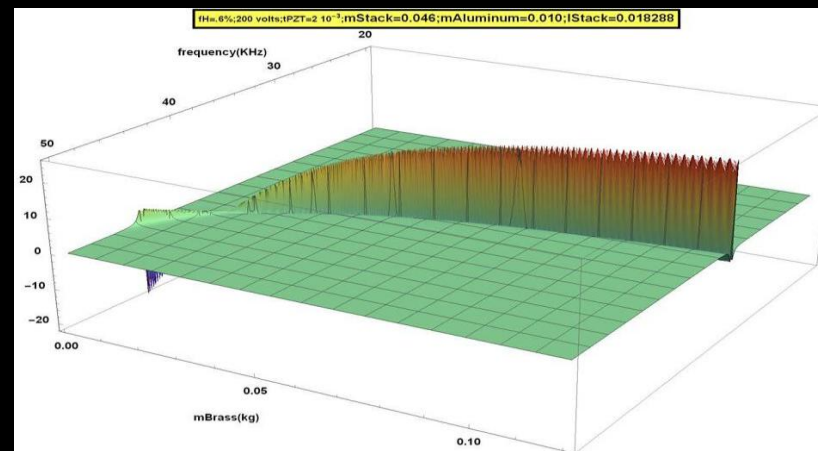
doubts if the observed effects are due to mass fluctuations.

# Discrepancy between models and experiments

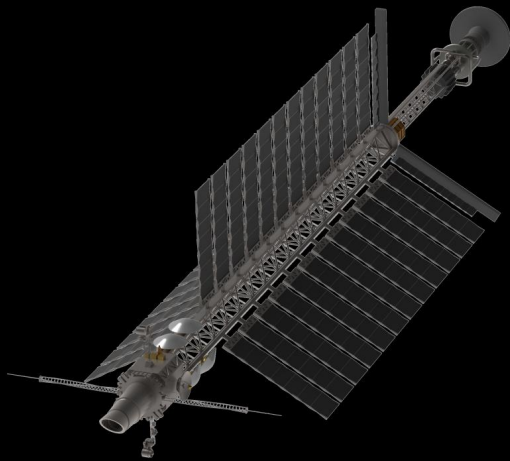
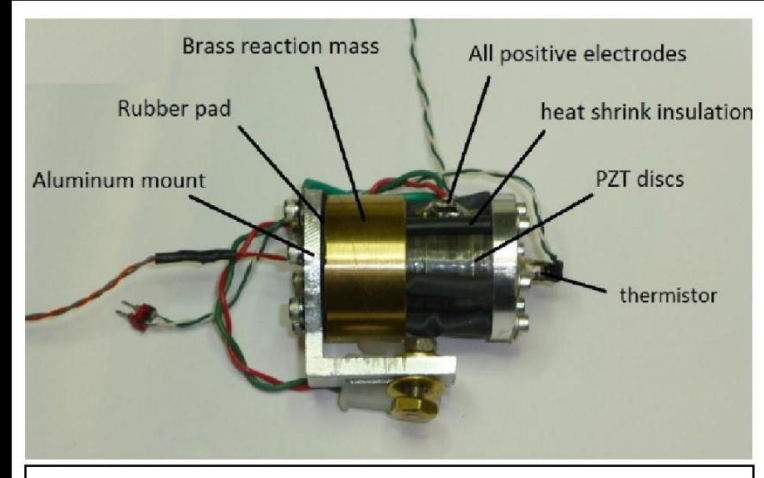


Rodal, J., 2016, "Mach Effect Propulsion, an Exact Electroelasticity Solution,"  
Estes Park Advanced Propulsion Workshop

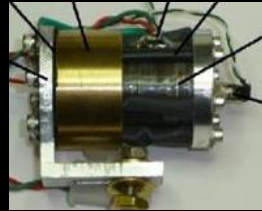
...the total non-dimensional coupling factor for the Mach effect force ... **is of the order of  $(10^{-2})^3 = 10^{-6}$** . The reason for the need of this coupling factor ...remains to be fully explored.



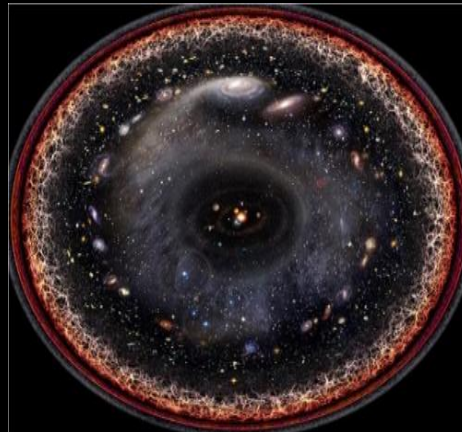
# 9 confusions in the literature



# Confusion in the literature



1. Confusing the *local* potential  $\phi$  ( $\sim 0$ ) with the *total universe's* potential  $\Phi$  ( $\sim c^2$ )



# Calculating the universe's potential $\Phi$

Sciama: a **solid ball** of radius  $R$ , uniform density

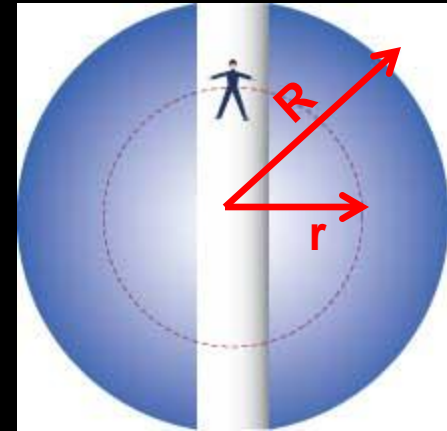
$$\Phi = U/m = - G \int (\rho / r) dV$$

$$V = (4\pi/3) r^3$$

$$\begin{aligned} \Phi &= - G \int (\rho / r) 4\pi r^2 dr \\ &\text{integrate between } r=0 \text{ and } r=R: \\ &= - 2\pi G \rho R^2 \end{aligned}$$

$$\text{replace } M = \rho V = \rho (4\pi/3) R^3$$

$$\Phi = - (3/2) G M / R$$



Sciama then drops factor of  $2\pi$  saying “it is approximate” and just calculates  $\phi = - 0.24 G M / R$   
Davidson calculates Schwarzschild radius formula

# Inside a **solid ball** of radius $R$ , uniform density

1) Potential energy proportional to  $r^2$

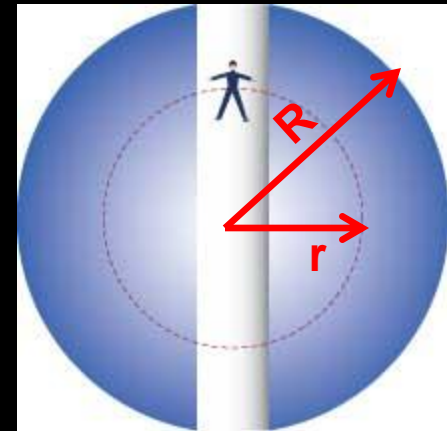
$$\Phi = U/m = - G M (3 R^2 - r^2)/(2 R^3)$$

at center  $r = 0$ ,  $\Phi = - (3/2) G M / R$

(like Sciama,

but there is no center and no edge!)

at periphery  $r = R$ ,  $\Phi = - G M / R$



2) **Gravitational acceleration** proportional to distance from center (like a spring):

$$a = F/m = d\Phi/dr = G M r / R^3$$

at center  $r = 0$ ,  $a(r=0) = 0$  (but there is no center and no edge!)

at periphery  $r = R$ ,  $\left[ \begin{array}{l} a(r=R) = G M / R^2 \\ \text{for universe} = a(r=R) = 5.14 \times 10^{-12} \text{ g} \sim 0 \end{array} \right.$



# Calculating the universe's potential $\Phi$

## Newton's (**hollow**) shell theorem

### 1) Constant potential energy

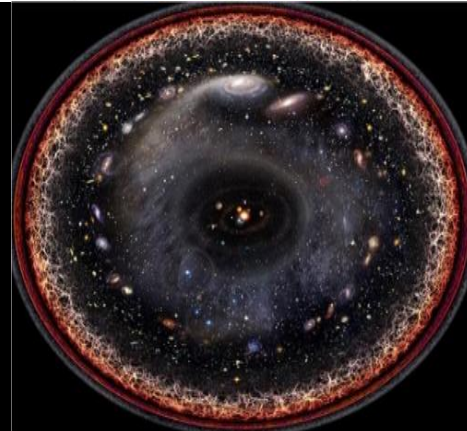
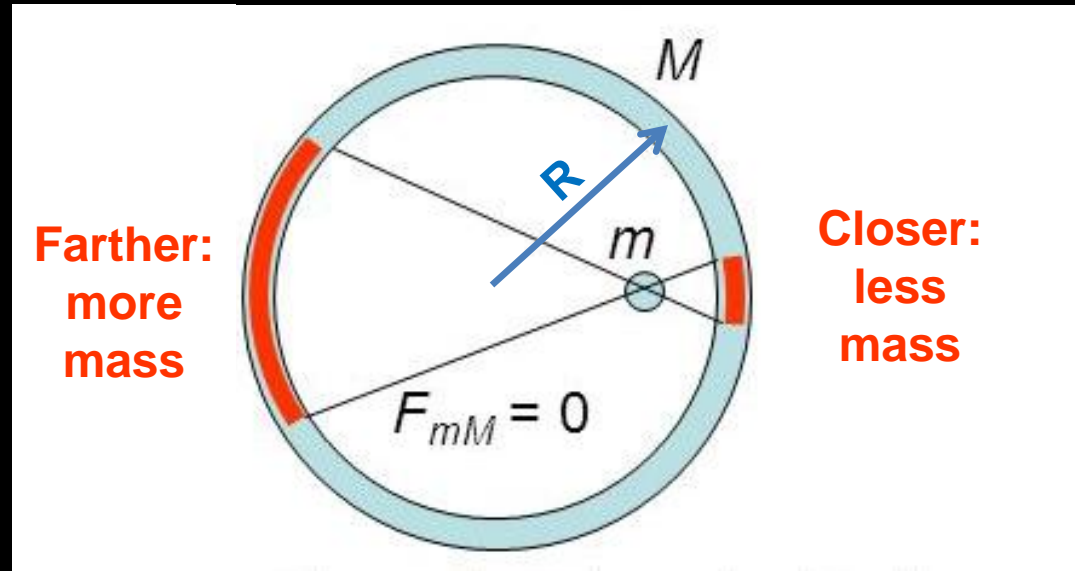
$$U = - G M m / R$$

everywhere inside the shell  
with radius  $R$ . **Constant**  
potential

$$\Phi = U/m = - G M / R$$

### 2) Zero gravitational force everywhere inside the shell:

$$a = F/m = d\Phi/dr = 0$$

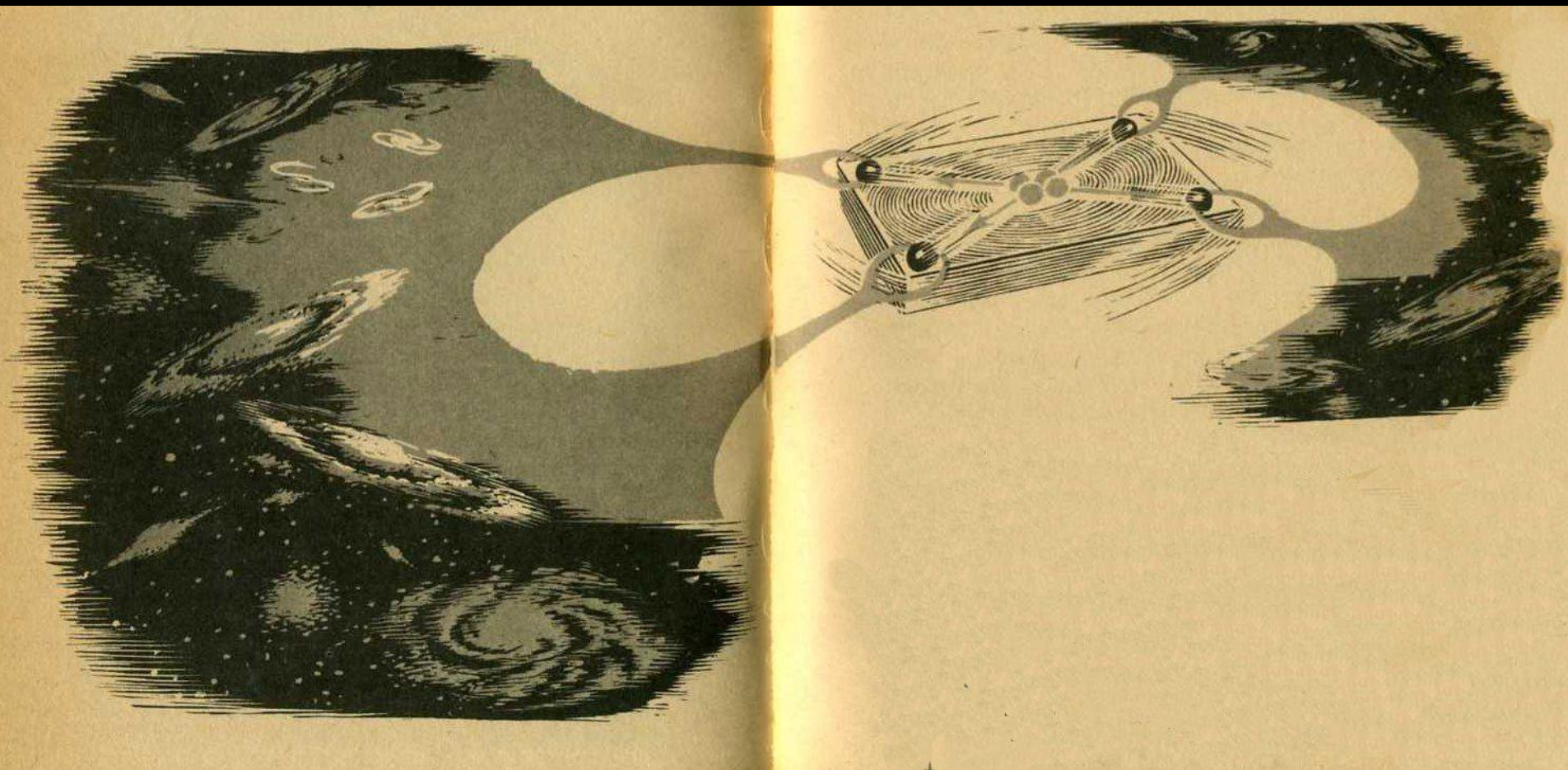




# Mach's origin of inertia

“mass-energy **there** rules inertia here.”

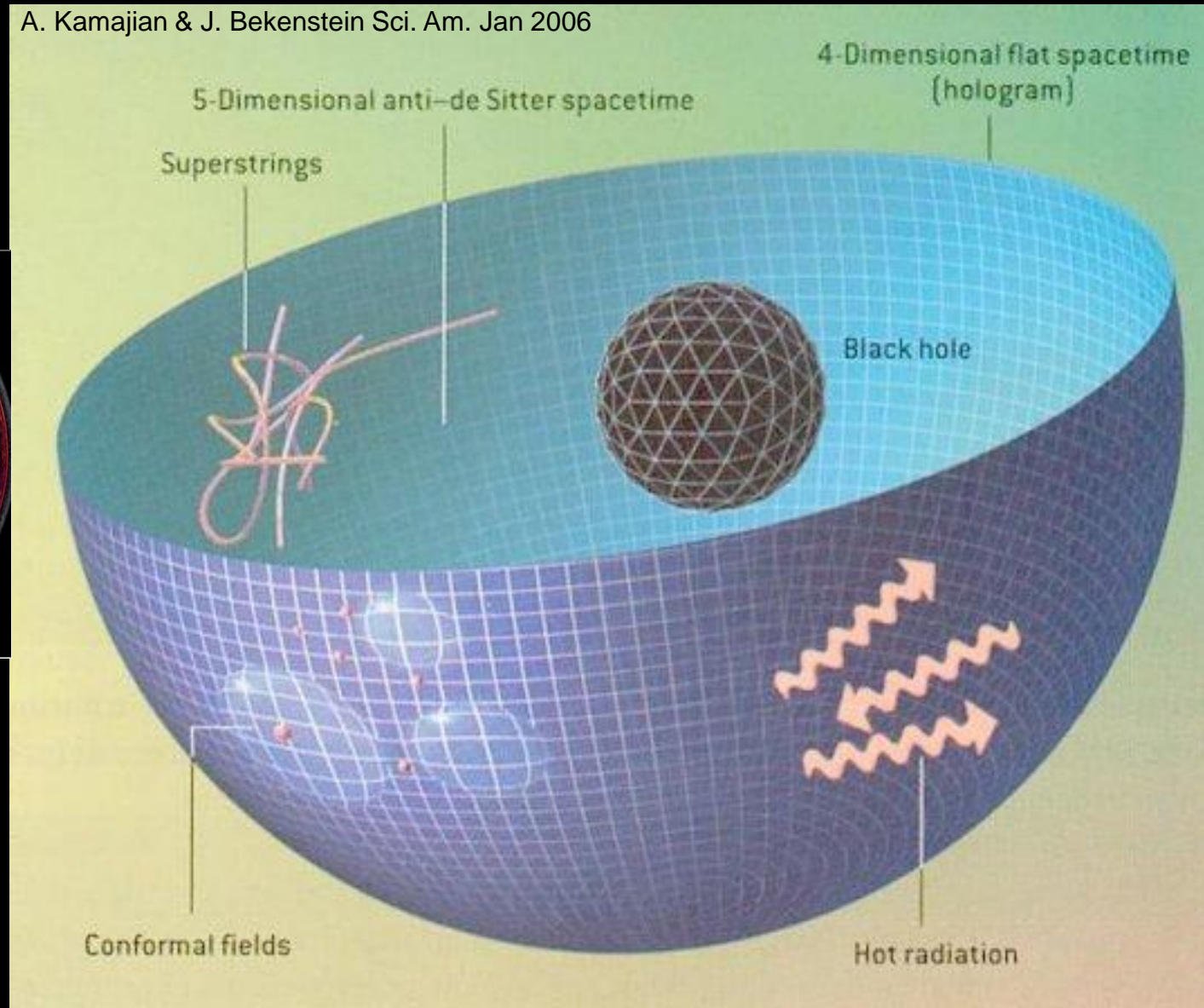
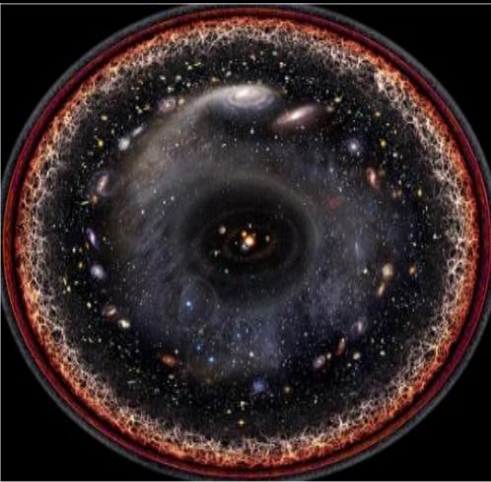
Ciufolini and Wheeler (1995) (p. 399)



(David E. Rowe, from “The Relativity Explosion,” 1976, Author: M. Gardner, Illustrator: A. Ravielli)

# The universe as a hologram (Susskind, etc.)

A. Kamajian & J. Bekenstein Sci. Am. Jan 2006





$\phi_g/c^2 \neq -1$  actually  $\phi_g/c^2 \sim 0$

To consider mass fluctuations, the last assumption must be dropped, allowing the following time-varying solution,

$$\nabla \bar{g} = -\Delta \phi_g = -4\pi G \rho_0 - \frac{1}{c^2} \frac{\partial^2 \phi_g}{\partial t^2} = -4\pi G \left( \rho_0 + \frac{1}{4\pi G c^2} \frac{\partial^2 \phi_g}{\partial t^2} \right) \quad (3)$$

where  $g$  is the gravitational field,  $\phi_g$  the gravitational potential and  $\rho_0$  the stationary mass

$$\delta \rho_0 = \frac{1}{4\pi G c^2} \frac{\partial^2 \phi_g}{\partial t^2} = \frac{\phi_g}{4\pi G c^2 m_0} \frac{\partial^2 m_0}{\partial t^2} = \frac{\phi_g}{4\pi G c^2 \rho_0} \frac{\partial^2 \rho_0}{\partial t^2} \quad (4)$$

where we used the simple gravitational scalar potential of the point mass  $m_0$ ,  $\phi_g = -G \cdot m_0 / r$ , to

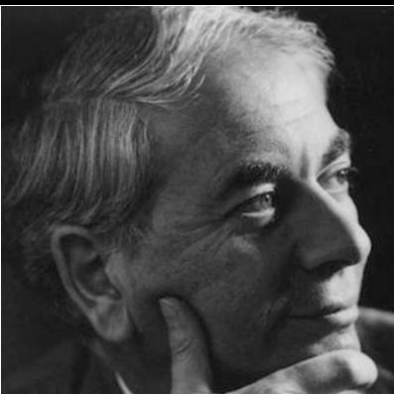
arrive at the Woodward mass formula [1]. Woodward then applied Sciama's inertia model [6],

where the effect of the surrounding mass of the universe follows  $\phi_g/c^2 = -1$ . Contrary to

Woodward, a negative sign is used as in Sciama's paper because a gravitational potential is **No**

Sciama clearly distinguished between the *local potential*  $\phi$  ( $\sim 0$ ) and the *total universe's potential*  $\Phi$  ( $\sim c^2$ ), and *used different notation for them*

Sciama, L., 1953, "On the Origin of Inertia," MNRAS, 113, 1, pp. 34-42



The total field at the particle is zero if

$$-\frac{M}{r^2} - \frac{\phi}{c^2} \frac{dv}{dt} = \frac{\Phi}{c^2} \frac{dv}{dt}$$

or

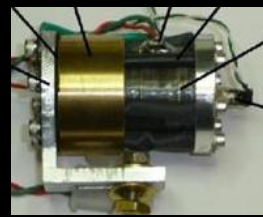
$$\frac{M}{r^2} = - \left( \frac{\Phi + \phi}{c^2} \right) \frac{dv}{dt}.$$

Furthermore, the gravitational constant satisfies the equation

$$\frac{\Phi + \phi}{c^2} = - \frac{1}{G}$$

or, since  $\phi \ll \Phi$  (*cf.* Section 4 (iii)),

$$G\Phi = -c^2.$$



Mega drive  $\phi/c^2 \sim 0$

	m (kg)	r (m)	$-\phi/c^2$	$v_e = \sqrt{-2\phi}$ (km/sec)
MEGA drive brass @ 0.018m	0.1	0.018	$4.1 \times 10^{-27}$	$2.7 \times 10^{-8}$
Earth @ surface	$6.0 \times 10^{24}$	$6.4 \times 10^6$	$7.0 \times 10^{-10}$	11
Sun @ Earth's orbit	$2.0 \times 10^{30}$	$1.5 \times 10^{11}$	$9.8 \times 10^{-9}$	42
Sun @ surface	$2.0 \times 10^{30}$	$7.0 \times 10^8$	$2.1 \times 10^{-6}$	618
Milky Way @ our Earth location	$2.0 \times 10^{41}$	$2.5 \times 10^{20}$	$1.7 \times 10^{-6}$	550
Black hole @ event horizon	Schwarzschild radius		0.5	c
Universe, ord. mat. horiz., WMAP crit. $\rho$	$1.5 \times 10^{53}$	$4.4 \times 10^{26}$	0.37	



Universe -  $\Phi/c^2 \sim 0.5$



Best estimate:  $-\Phi/c^2 \sim 0.4$  (similar to black hole  $-\phi/c^2 \sim 0.5$  )  
including dark matter  $-\Phi/c^2 \sim 2.4$

Universe, ord. mat. horiz., WMAP crit.dens.

Black hole @ event horizon

Milky Way @ our Earth location

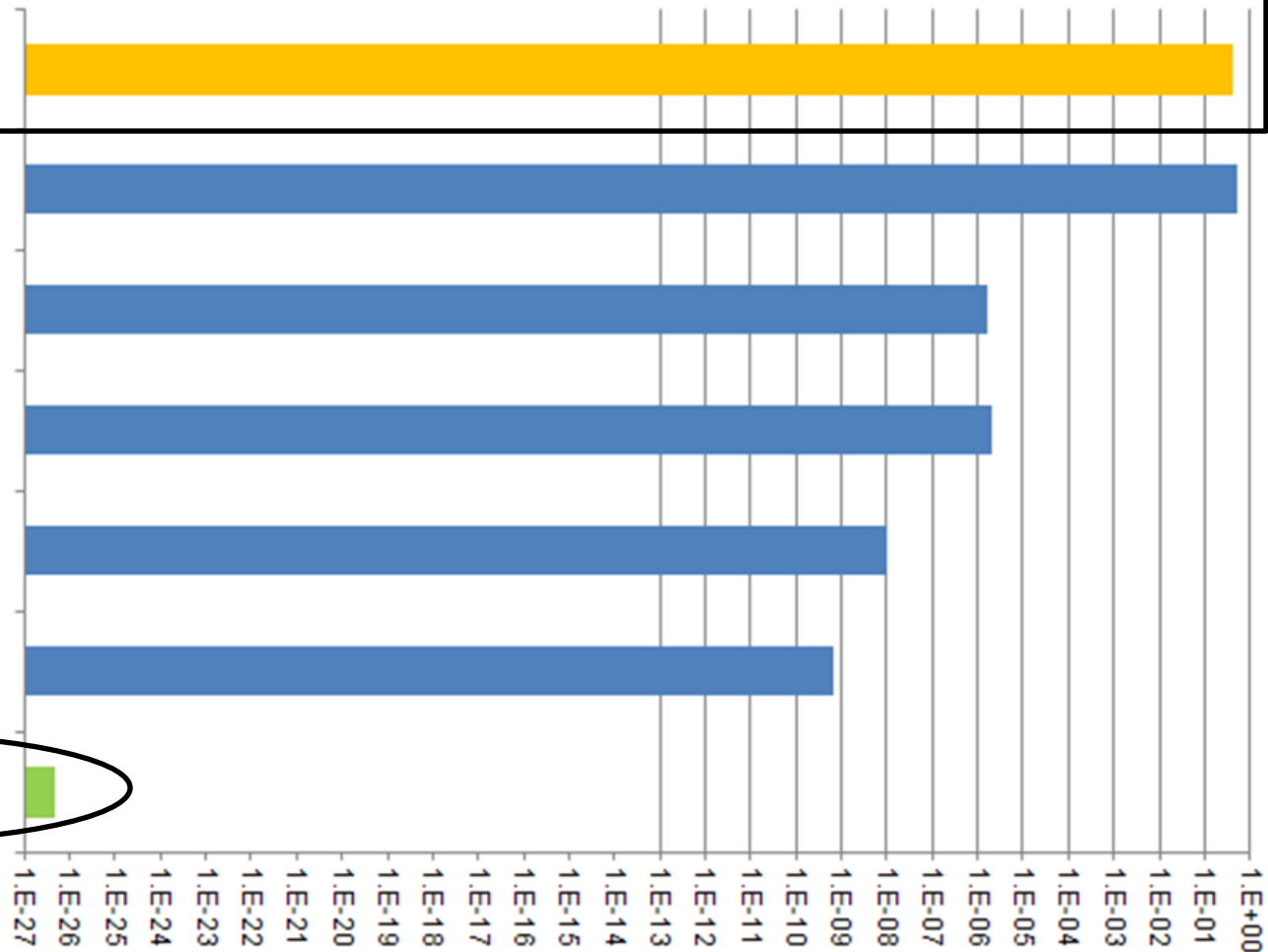
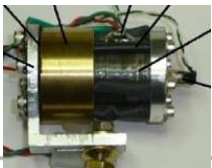
Sun @ surface

Sun @ Earth's orbit

Earth @ surface

MEGA drive brass @ 0.018m

$\phi/c^2 \sim 0$

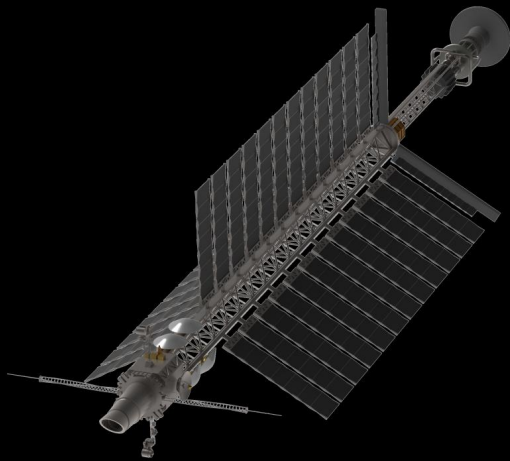


Dimensionless potential energy:  $-\phi/c^2$   
(logarithmic<sub>10</sub> scale)

# Confusion in the literature



## 2. Where is the Woodward effect?





# Where is the Woodward effect ?



Let us put this together, setting  $h_{00} \equiv -2\phi/c^2$ , to accord with the usual identification of the Newtonian potential  $\phi$  with the time-time component of the metric perturbation. Then the linear field equation (1) can be written:

$$\nabla^2 \phi - \frac{\partial^2 \phi}{c^2 \partial t^2} = \frac{4\pi G}{c^2} \rho W_0^2 \quad (6)$$

Equation (7) allows us to write the time derivative of the field in terms of particle energy:

$$\frac{\partial^2 \phi}{\partial t^2} = \frac{\partial}{\partial t} \left( \frac{c^2}{p^0} \frac{\partial p^0}{\partial t} \right) = \frac{c^2}{p^0} \frac{\partial^2 p^0}{\partial t^2} - \left( \frac{c}{p^0} \right)^2 \left( \frac{\partial p^0}{\partial t} \right)^2 \quad (8)$$

so that the field equation (6) now becomes:

$$\nabla^2 \phi = \frac{4\pi G}{c^2} \rho W_0^2 + \frac{1}{p^0} \frac{\partial^2 p^0}{\partial t^2} - \left( \frac{1}{p^0} \right)^2 \left( \frac{\partial p^0}{\partial t} \right)^2 \quad (9)$$

This is strikingly similar to (VI-5). To make the identification complete, put  $p^0 = mc$  and  $W^0 = c$ , and convert the mass factors to mass density:

$$\nabla^2 \phi = \frac{4\pi G}{c^2} \rho W_0^2 + \frac{1}{\rho} \frac{\partial^2 \rho}{\partial t^2} - \left( \frac{1}{\rho} \right)^2 \left( \frac{\partial \rho}{\partial t} \right)^2 \quad (10)$$

Furthermore, this recovers the 2nd and 3rd terms on the RHS of (VI-5) if Jim's original substitution is used in those terms to set ratios of  $\phi/c^2 \rightarrow 1$ . Missing from (10) relative to Jim's (VI-5) are the quadratic time derivatives of the field. Neither of those terms are important to Jim's Mach effect, so it appears they are dispensable parts of his theory. I like this simple derivation here because it reproduces the essential parts of (VI-5) without any assumptions or without the uncertainty of where to substitute for  $c^2$ .

No !

?

# Where is the Woodward effect ?

Not here!

These terms are the Woodward effect !



$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = 4\pi G \rho_0 + \frac{\phi}{\rho_0 c^4} \frac{\partial^2 E_0}{\partial t^2} - \left( \frac{\phi}{\rho_0 c^4} \right)^2 \left( \frac{\partial E_0}{\partial t} \right)^2 - \frac{1}{c^4} \left( \frac{\partial \phi}{\partial t} \right)^2. \quad (7)$$

This is a classical wave equation for the gravitational potential  $\phi$

Woodward, J., 2004, "Flux Capacitors," Foundations of Physics, 34, 10, pp. 1475-1514

Notation should have differentiated between  $\Phi/c^2 \sim -1$ , and  $\phi \sim 0$

# Where is the Woodward effect ?



why not  
here?

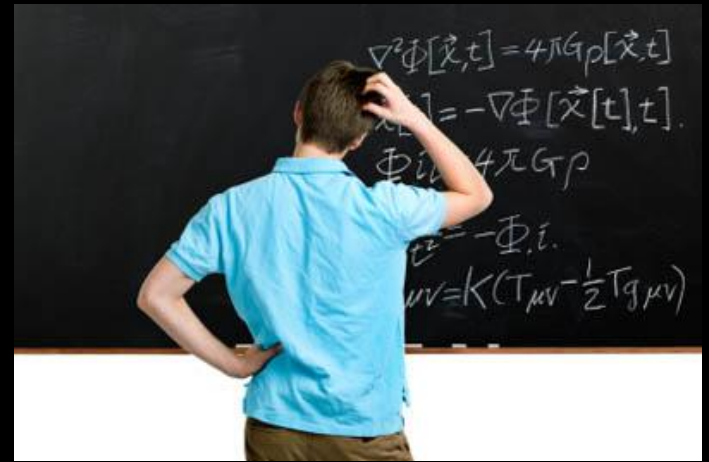
$$\nabla^2\phi - \frac{1}{c^2} \frac{\partial^2\phi}{\partial t^2} = 4\pi G\rho_0$$

= 0

• Woodward considered wave effect with **D'Alembertian instead of just static Laplacian**  $\nabla^2\phi$

- For  $\rho = 0$  (no mass source) Einstein's equations have **vacuum solutions** (Ricci flat but not Riemann flat)
- Vacuum solutions are Non-Machian: e.g. **anti-Machian** Ozsváth–Schücking metric : stationary, singularity-free, not isometric with Minkowski metric
- Energy fluctuations in spacetime **without any mass source**

# Confusion in the literature



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3. Cannot have a 1-D model for Mach propulsion!

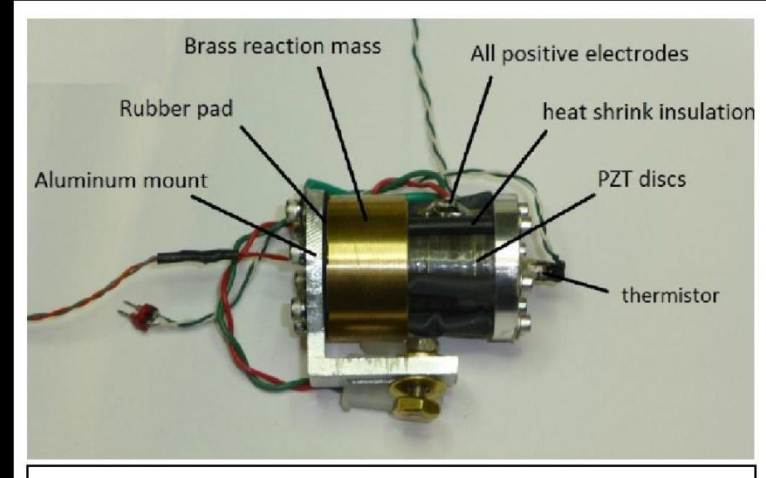
# Cannot have a 1-D model for Mach propulsion

- Cannot have a clamped boundary condition in space
- Cannot push or pull something with internal forces (electrostriction, piezoelectricity, etc.) Violation of conservation of momentum!



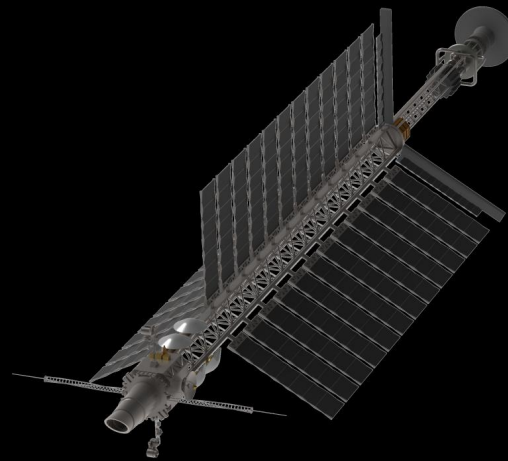
- A 1-D fluctuating mass will not accelerate in any direction

# Confusion in the literature



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4. Can't have changes in mass due to energy fluctuations, without damping



# Simple Harmonic Motion: no fluctuation in total energy!

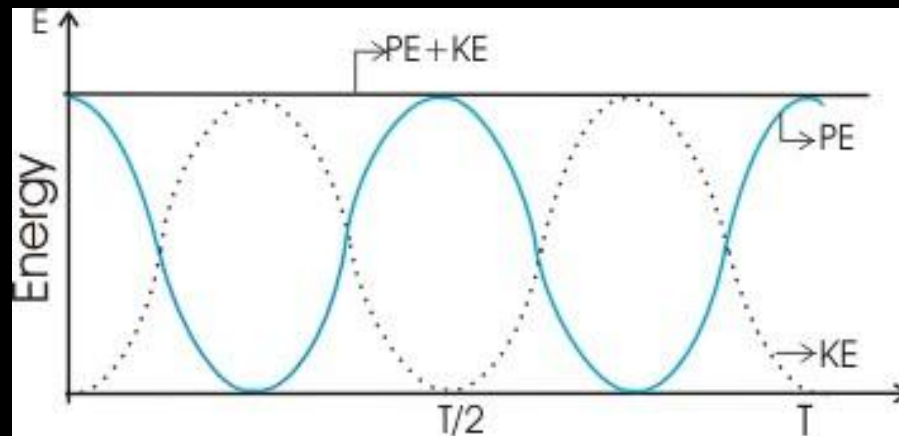
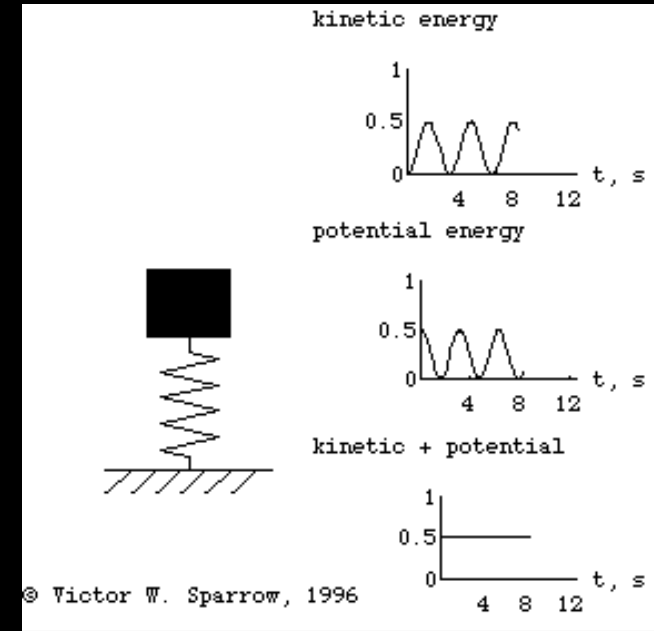
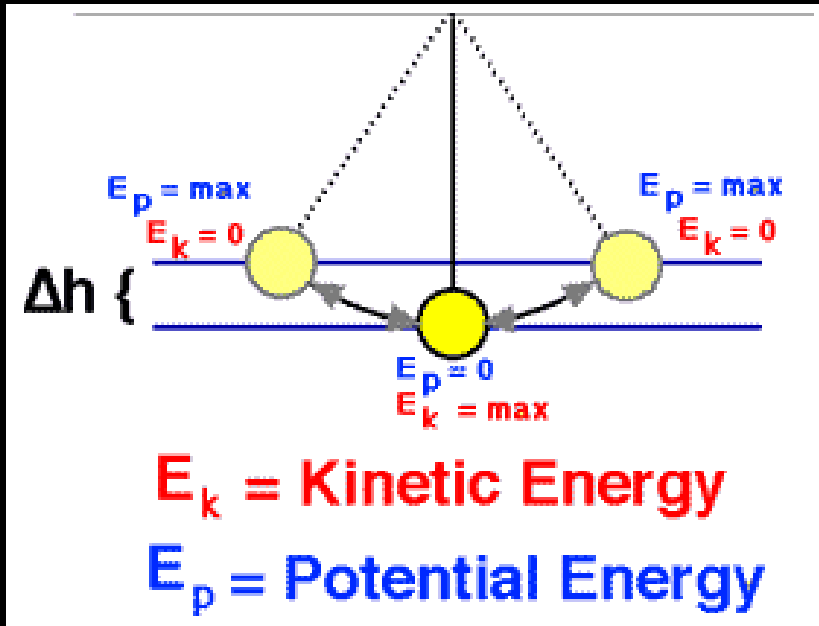
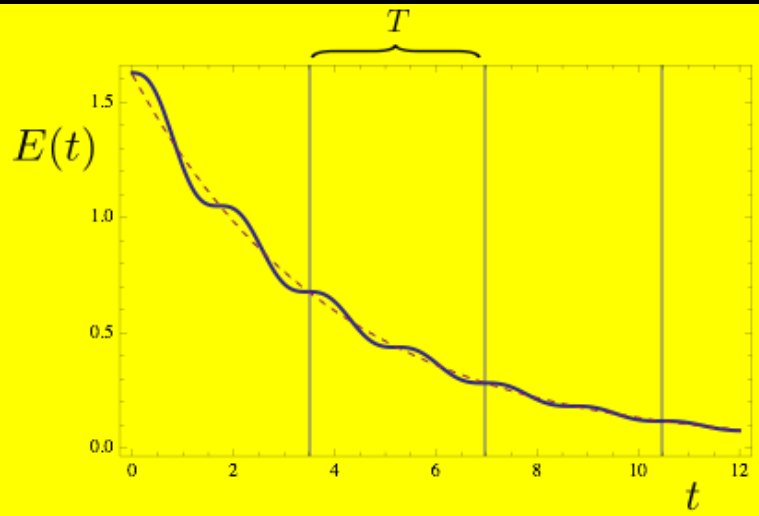


Figure 5:- Energy exchange in SHM



Energy in the (under) **damped** oscillator:  
total **energy fluctuation** only for **damping**  $> 0$



Energy-mass fluctuation is only possible with damping

$$\gamma = 2 \zeta \omega_0 = c/m > 0$$

hence the total energy is

$$\begin{aligned} E &= E_k + E_p = \frac{1}{8} m A^2 e^{-\gamma t} \left( [\gamma \cos(\omega_d t + \phi) + 2\omega_d \sin(\omega_d t + \phi)]^2 + 4\omega_0^2 \cos^2(\omega_d t + \phi) \right) \\ &= \frac{1}{8} m A^2 e^{-\gamma t} (4\omega_0^2 + 2\gamma\omega_d \sin(2\omega_d t + 2\phi) + \gamma^2 \cos(2\omega_d t + 2\phi)) \\ &= \frac{1}{2} m A^2 \omega_0^2 e^{-\gamma t} \left[ 1 + \frac{\gamma}{2\omega_0} \cos(2\omega_d t + \phi') \right], \end{aligned}$$

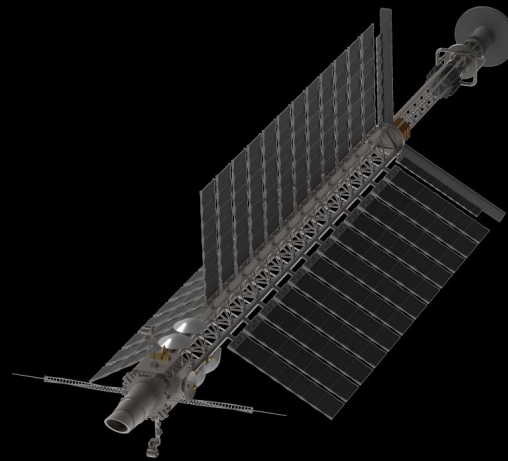
and  $\phi' = -\tan^{-1}(2\omega_d/\gamma)$ .

Alexei Gilchrist

# Confusion in the literature

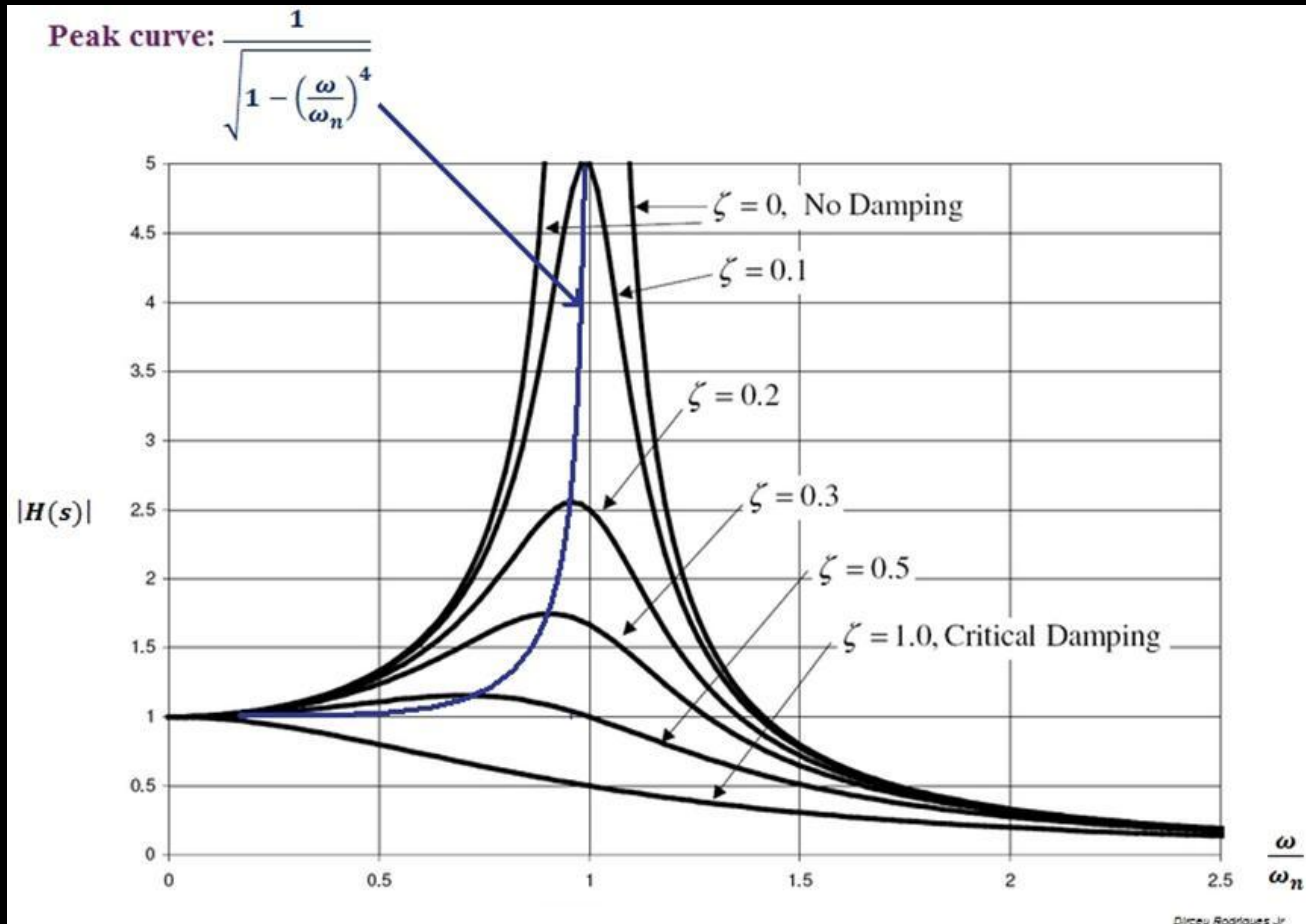


5. Models that ignore damping cannot realistically predict frequency  $\omega$  dependence



# Confusion in the literature

- MEGA drive operates at resonance. Amplitude at resonance is governed by damping (No damping = INFINITE amplitude)



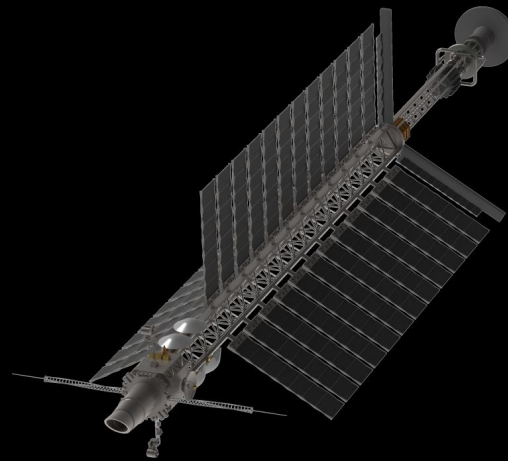
# Confusion in the literature

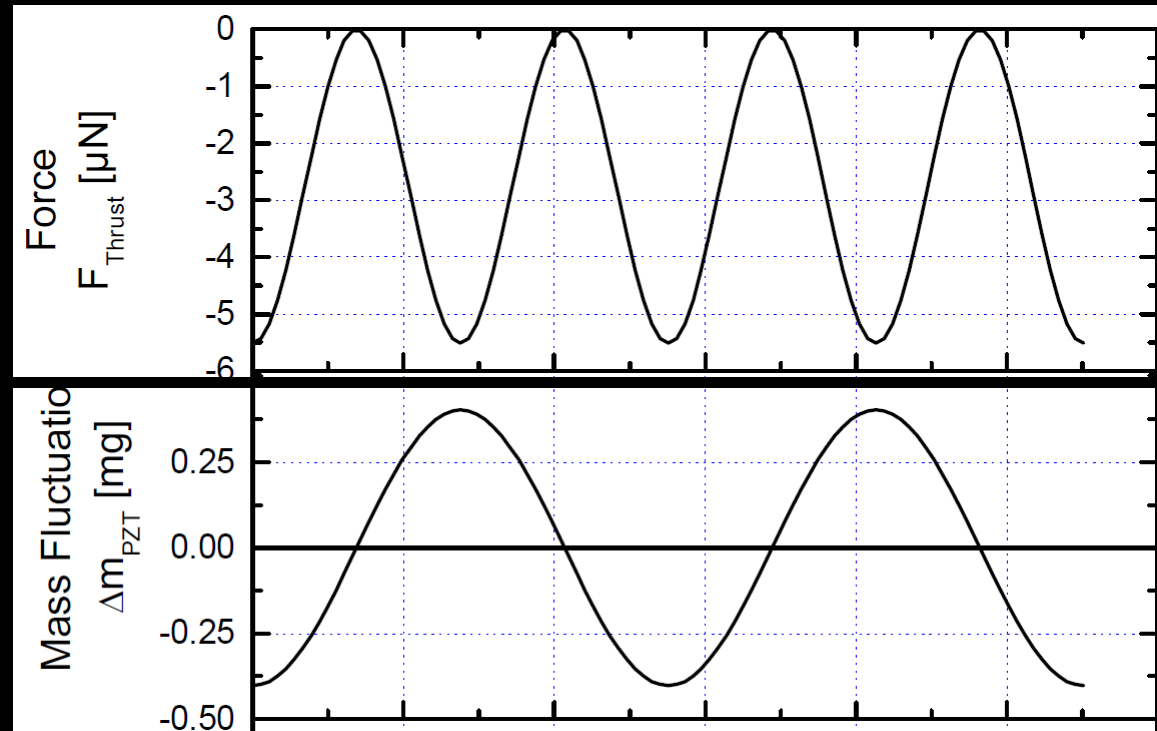
- unphysical to predict  $\omega^6$  or  $\omega^4$  dependence when ignoring damping at resonance:
  - heat generation is a function of frequency
  - higher frequency modes are more heavily damped

# Confusion in the literature



## 6. Mass fluctuation predictions that are incompatible with physical experimental data

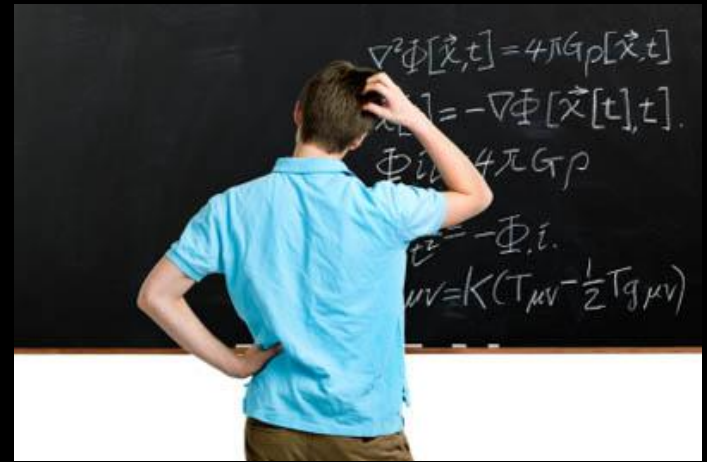




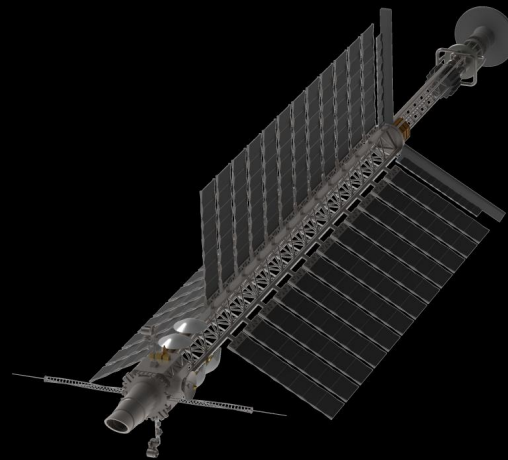
The amplitude is close to 0.4 mg which is a huge value

Mass fluctuation has to be compatible with existing dynamic physical data

# Confusion in the literature



7. Mechanical-energy is not the only type of energy that gravitates or that has a gravitational potential





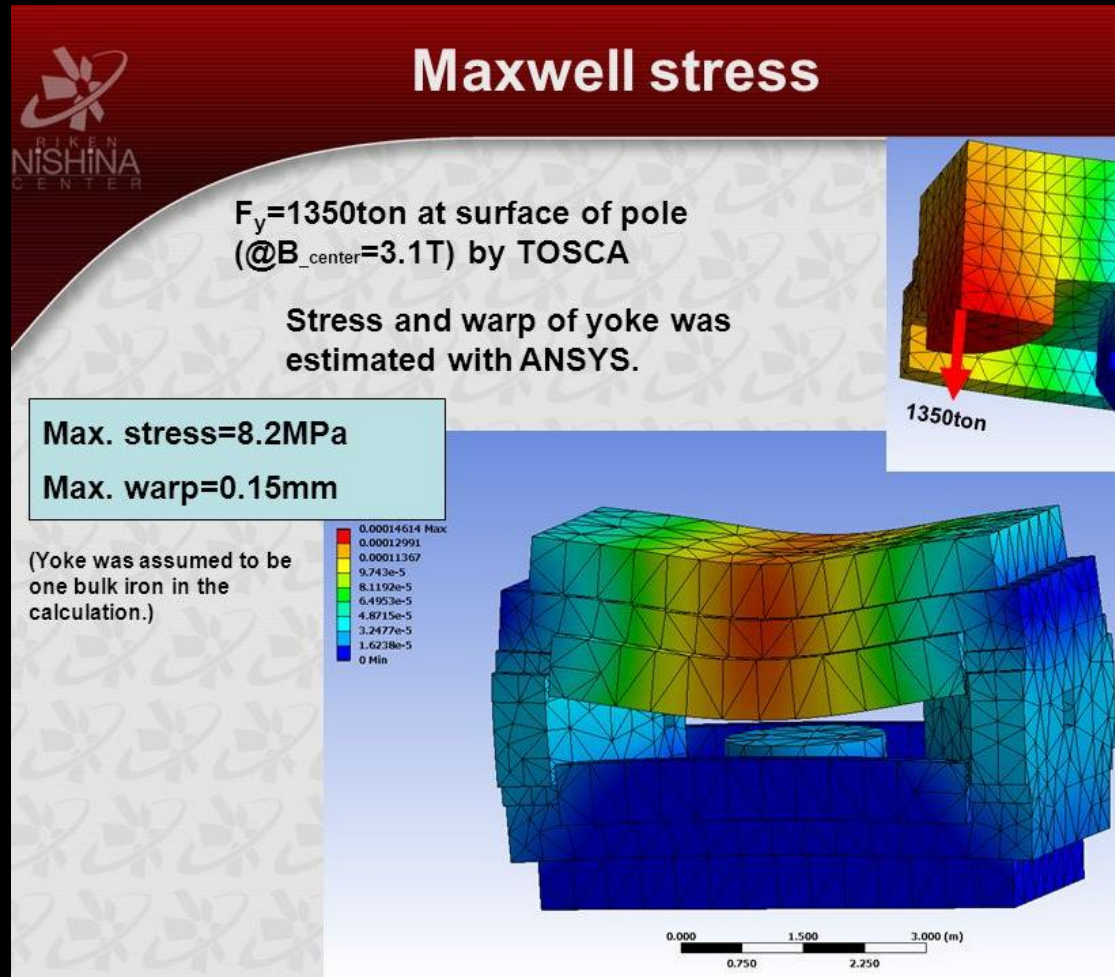
In general relativity all types of energy-momentum gravitate!

Stress energy, kinetic energy, electromagnetic energy, thermal energy, etc., they all gravitate, they all have a potential.

$$T^{\mu\nu} = \begin{bmatrix} \frac{1}{2} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) & S_x/c & S_y/c & S_z/c \\ S_x/c & -\sigma_{xx} & -\sigma_{xy} & -\sigma_{xz} \\ S_y/c & -\sigma_{yx} & -\sigma_{yy} & -\sigma_{yz} \\ S_z/c & -\sigma_{zx} & -\sigma_{zy} & -\sigma_{zz} \end{bmatrix}$$

$E=mc^2$  The total energy  $E$  is conserved

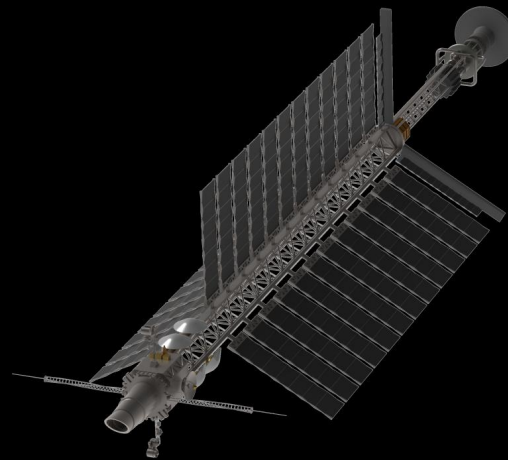
*Mechanical* stress energy **is not** privileged in Mach's principle!



# Confusion in the literature



8. “m” term in Hoyle-Narlikar is not a local mass source!





Not here: m is not a mass

The mass is here in  $T_{\alpha\beta}$  !

In the conformal theory of Hoyle and Narlikar [19] , the smooth fluid approximation alone leads to the field equation;

$$\frac{1}{2}m^2 \left( R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R \right) = -3T_{\alpha\beta} + m(g_{\alpha\beta}g^{\mu\nu}m_{;\mu\nu} - m_{;\alpha\beta}) + 2(m_{;\alpha}m_{;\beta} - \frac{1}{4}m_{;\gamma}m^{;\gamma}g_{\alpha\beta}) \quad (1)$$

Taking the Christoffel symbols (of the covariant derivatives) to be zero, and using  $c = 1$  for consistency,

$$\frac{m^2}{2}\delta M = \frac{2m}{3} \left( \frac{\partial^2 m}{\partial t^2} \right) - \frac{1}{2} \left( \frac{\partial m}{\partial t} \right)^2 \quad (5)$$

when we divide by  $m^2/2$  (which is multiplied throughout in Eq. (1) ) we get mass fluctuation terms as follows,

$$\delta M = \frac{4}{3m} \left( \frac{\partial^2 m}{\partial t^2} \right) - \frac{1}{m^2} \left( \frac{\partial m}{\partial t} \right)^2 \quad (6)$$

Apart from a 4/3 numerical factor these are the mass fluctuation terms, originally derived by one of us JFW

not

The mass is here in  $T_{\alpha\beta}$

In the conformal theory of Hoyle and Narlikar [19], the smooth fluid approximation alone leads to the field equation;

$$\frac{1}{2}m^2 \left( R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R \right) = -3T_{\alpha\beta} + m(g_{\alpha\beta}g^{\mu\nu}m_{;\mu\nu} - m_{;\alpha\beta}) + 2(m_{;\alpha}m_{;\beta} - \frac{1}{4}m_{;\gamma}m^{;\gamma}g_{\alpha\beta}) \quad (1)$$

Fearn et.al., 2015, "Theory of a Mach Effect Thruster II," JMP

m is not a mass

- Hoyle-Narlikar's smooth-field ~ Brans-Dicke's
- m is a scalar **field** pervading all of spacetime and its associated particle has zero mass
- m is only due to the inverse square root of G

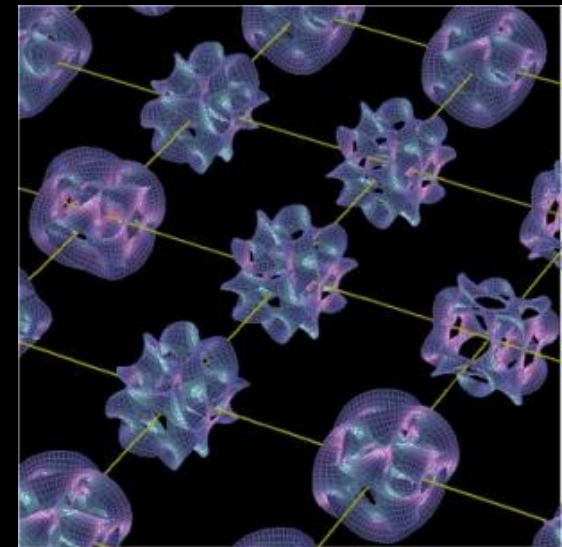
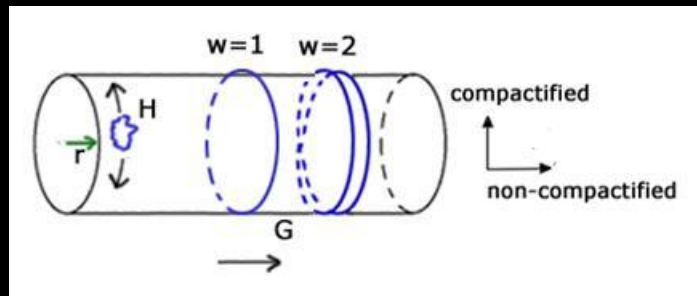
$$m = c^2 / \sqrt{[(4 \pi / 3) G]} \sim c^2 / (2\sqrt{G})$$

- $m_{\text{planck}} = \sqrt{[\hbar c / G]}$

*“I would very strongly doubt that [the Hoyle-Narlikar particle field equations] have any [mathematical] solutions. Maybe the [Hoyle-Narlikar] theory should be taken seriously only after you have gone to the fluid average.”*

Jürgen Ehlers \*

- The fluid average version of Hoyle-Narlikar (HN) is a conformal scalar-tensor gravitation theory, similar to Jordan-Brans-Dicke's theory (JBD)
- Jordan-Brans-Dicke's theory is much more studied, with several exact solutions (unlike HN), and is derivable from Kaluza-Klein cosmology (after compactification, etc.)



- A gravitational scalar field as in JBD is an unavoidable feature of superstring, supergravity and M-theory (string dilaton, etc.)
- **Coupling constant** ( $\omega$ ) in JBD, need one in HN too

# Jordan-Brans-Dicke's scalar-tensor theory

The field equations of the Brans/Dicke theory are

$$\square\phi = \frac{8\pi}{3+2\omega}T$$

$$G_{ab} = \frac{8\pi}{\phi}T_{ab} + \frac{\omega}{\phi^2}(\partial_a\phi\partial_b\phi - \frac{1}{2}g_{ab}\partial_c\phi\partial^c\phi) + \frac{1}{\phi}(\nabla_a\nabla_b\phi - g_{ab}\square\phi),$$

where

$\omega$  is the dimensionless Dicke coupling constant;

$$G_{ab} = R_{ab} - \frac{1}{2}Rg_{ab}$$

$\phi$  is the scalar field

Brans-Dicke scalar field goes like  $1/G$

$$\phi_{BD} = (2\omega+4) c^4 / [G (2\omega+3)]$$

Cassini–Huygens (C. Will 2014) shows coupling parameter  $\omega > 43,000$

hence

$$\phi_{BD} = c^4 / G \\ \sim 4 \text{ m}^2$$



# Jordan-Brans-Dicke's scalar-tensor theory

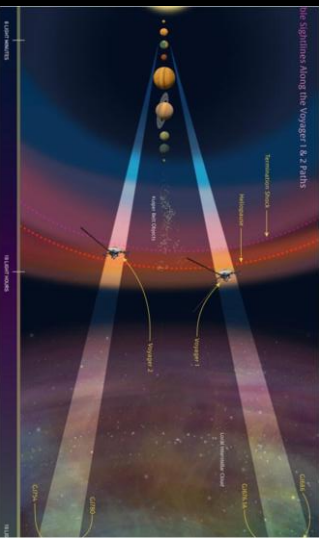
## Uncertainty on scalar coupling *in interstellar space*

- *Experimental* bounds on BD coupling  $\omega$  are from experiments (Cassini–Huygens, etc.) in our **solar system** that may not apply in interstellar space because of the chameleon effect (Khoury et.al.)

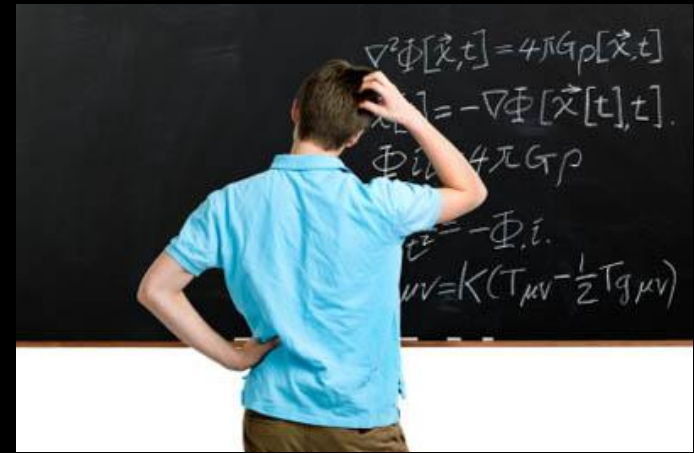
- Chameleon effect depends on the background energy density of the environment

- Nagata, Chiba, Sugiyama (PRD 2004): WMAP temperature power spectrum constraints  $10 < \omega < 10^7$  [*small coupling*]

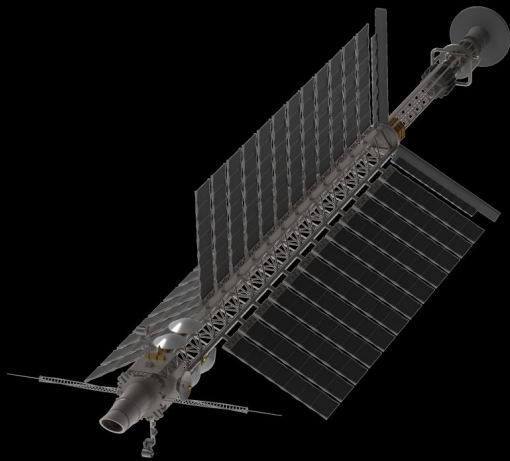
- Hrycyna, Szydlowski, Kamionka (PRD 2014): distant supernovae type Ia, and Hubble function  $H(z)$  measurements (using *Bayesian methods*), find  $-2.38 < \omega < -0.86$  [*large coupling*] in correspondence with low-energy limit of string theory  $\omega = -1$ . → MACH EFFECT LARGER IN INTERSTELLAR SPACE



# 9 confusions in the literature



9. Dissonance: repeating Wheeler's “mass-energy  
**there** rules inertia here”



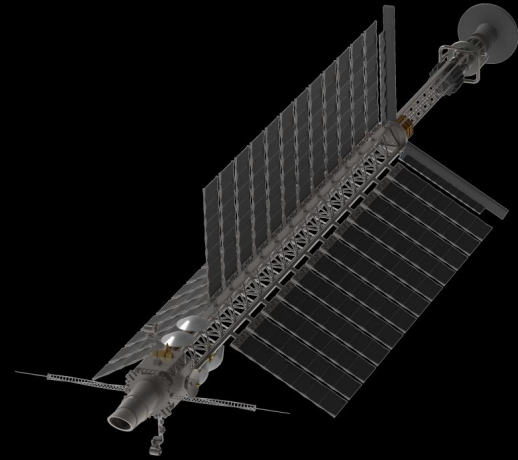
Ciufolini and Wheeler (1995) (p. 399)

Mass  $M_u = 10^{53} \text{ kg} =$

“there”  
rules inertia  
here

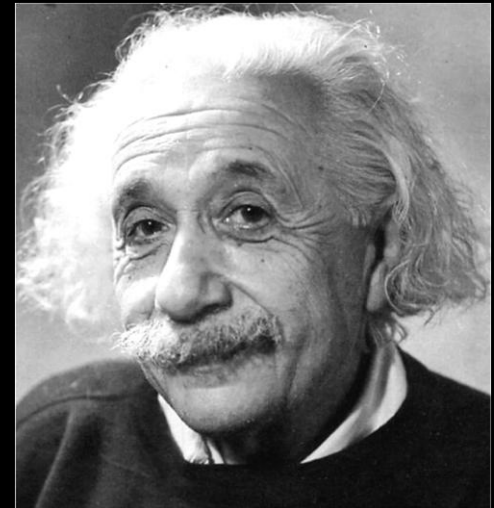
Rodal 10/2017

# Correct formulation



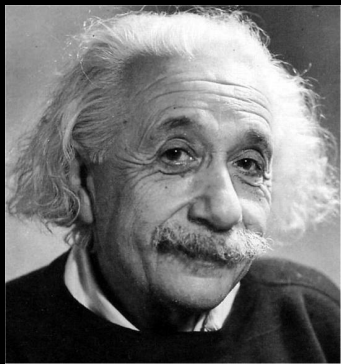
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1. Can one find the Woodward effect terms in Einstein's General Relativity?



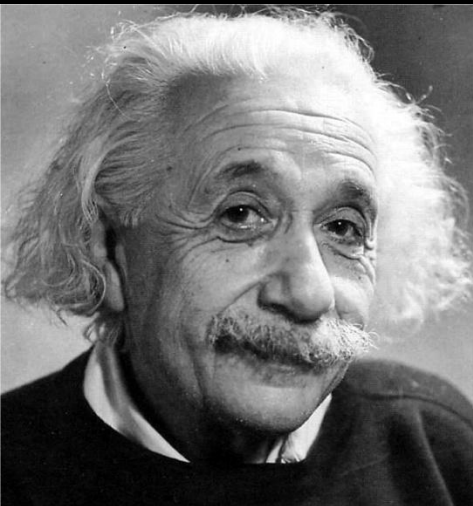
# Woodward effect terms in Einstein's General Relativity?

the time-time perturbation component of this term  $(c^4/4\pi G)h^{00}\frac{\partial^2 h^{00}}{c^2\partial t^2}$ , for the expansion in the near zone, to be of order 3PN (3rd post-Newtonian). For a very weak spherically symmetric gravitational field  $\phi = -Gm/r$ , upon replacing  $h^{00} = -2\phi/c^2$  this term becomes equal to  $(\phi/\pi G)\frac{\partial^2 \phi}{\partial t^2}$  which for variable mass  $m$ , constant  $G$  and constant position gives a term proportional to the second time derivative of the variable mass:  $-(\phi/\pi r)\frac{\partial^2 m}{\partial t^2}$ .



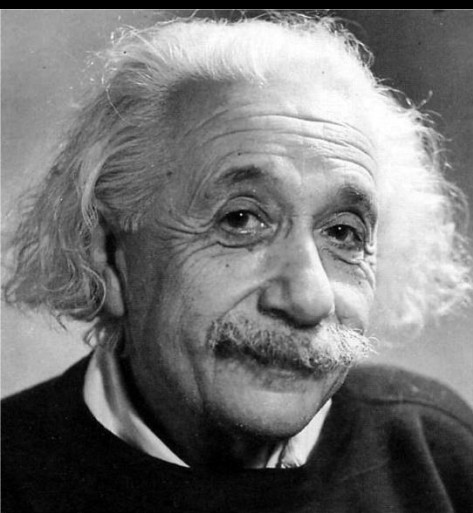
# Woodward effect terms in Einstein's General Relativity?

- The term in GR is 3 PN: infinitesimal, because the prefactor is the potential  $\phi$  due to the *local mass* instead of the entire universe's potential  $\Phi$
- GR gauge dependence: coordinate dependence.
- physical meaning is tied to a metric solution to the entire universe. GR admits anti-Machian solutions (Gödel, Ozsváth–Schücking) as well as Machian solutions (Friedman-Robertson-Walker).





Using the Bianchi identities... more promising but more complex...

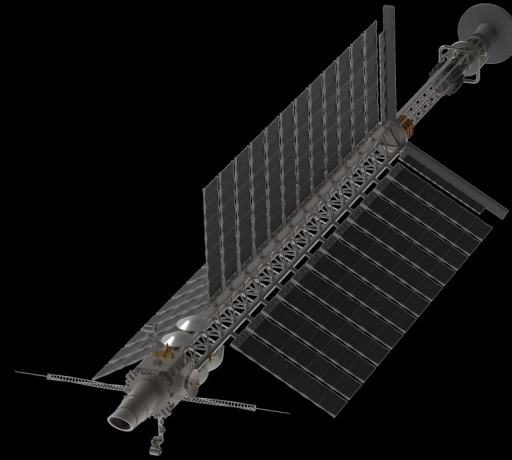


For a very weak gravitational field, in the linear limit, when the curvature is small so that quadratic terms in the Riemann curvature expression can be ignored, this equation reduces to:

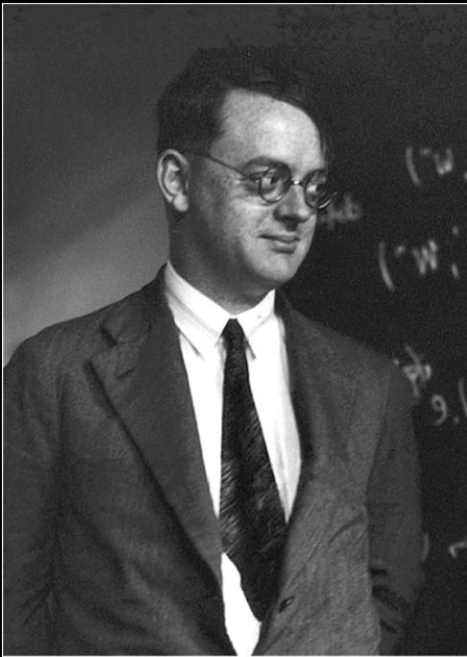
$$\begin{aligned}
 M \square R_{bc mn} &= \frac{8\pi G}{c^4} \left( (\overline{T_{nc}})_{,m,b} - (\overline{T_{mc}})_{,n,b} - (\overline{T_{nb}})_{,m,c} \right. \\
 &\quad \left. + (\overline{T_{mb}})_{,n,c} \right) \\
 \overline{T_{ij}} &= T_{ij} - \frac{1}{2} \eta_{ij} T \\
 T &= \eta^{rs} T_{rs} = T^r_r
 \end{aligned}
 \tag{8}$$

where I use Latin indices, for direct comparison with Padmanabhan (2010), and where these indices can be raised and lowered with the Minkowski metric  $\eta_{ij}$ . This equation is remarkable because it is gauge invariant (under infinitesimal coordinate transformations): it provides a gauge-independent description of propagating Riemann curvature waveforms.

# Correct formulation



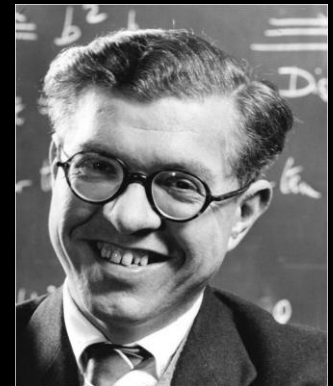
## 2. Scalar-tensor theories



Jordan



Brans



Hoyle



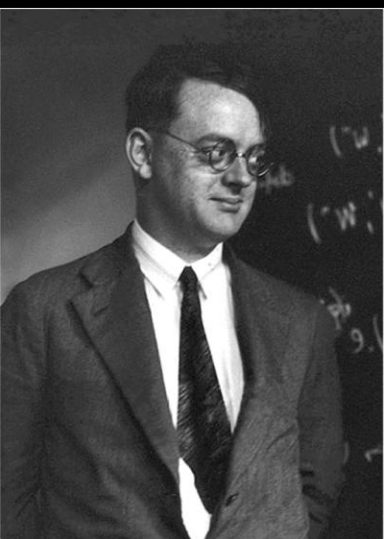
Dicke



Narlikar



**Invariant** form  
(gauge  
independent)  
of  
HN equations  
**valid for arbitrarily  
large gravitational  
field:**  
**all nonlinear  
scalar terms  
disappear**

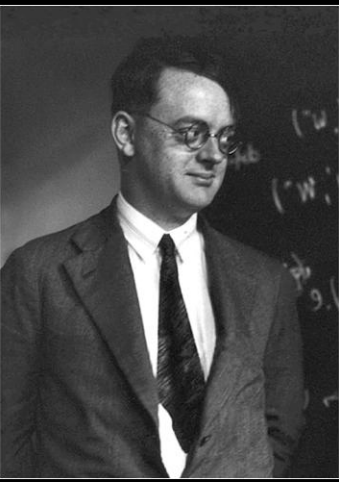


with the semicolon representing covariant differentiation, the smooth-fluid version of Hoyle-Narlikar's field equation is a scalar-tensor field theory of gravitation:

$$\begin{aligned} \frac{1}{2}m^2 \left( R_{ik} - \frac{1}{2}g_{ik}R \right) = & -3T_{ik} \\ & + m \left( g_{ik}g^{pq}m_{;pq} - m_{;ik} \right) \\ & + 2 \left( m_{;i}m_{;k} - \frac{1}{4}m_{;l}m^{;l}g_{ik} \right) \end{aligned} \quad (9)$$

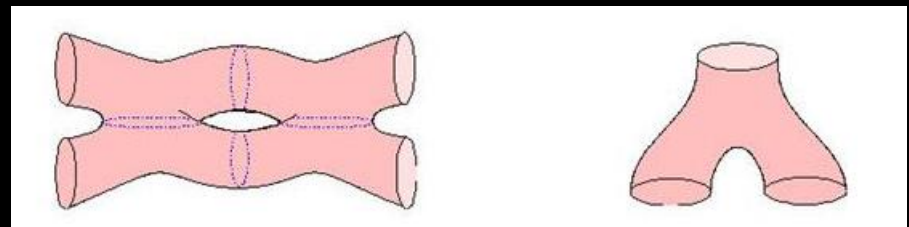
$$-R = \frac{8\pi G}{c^4}T + 6\frac{\square m}{m} \quad (12)$$

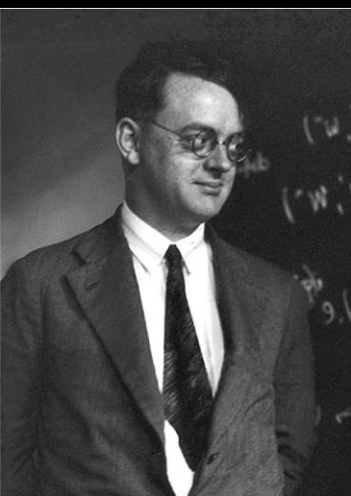
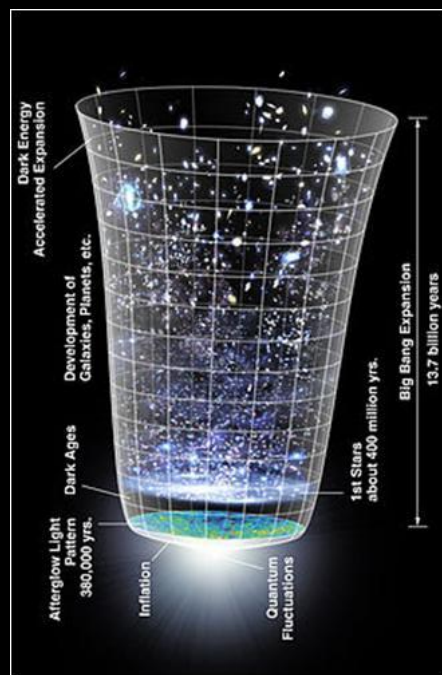
the scalar invariant form of HN's field equation for the universe's fluid mass



If the stress-energy-momentum scalar is zero  $T = 0$ , for example for a fluid with equation of state  $p = \rho c^2/3$ , the resulting equation is  $6\Box m + Rm = 0$  which can be interpreted as giving rise to hypothetical particles associated with the scalar field  $m$  called dilatons (p. 96 of Hoyle and Narlikar (1974)). In Brans-Dicke's gravitational theory if the gravitational constant  $G$  is allowed to become a dynamic field, the resulting particle from this field is also a dilaton. Dilatons also appear in theories with extra dimensions  $> 4$  when the volume of the compactified dimension is allowed to vary. It is a generic name for the Goldstone Boson associated with spontaneous breaking of scale invariance. In string theory, the dilaton is a closed-string, massless, spin-zero particle.

Conformal transformation:  
*dilational* invariance





$$-R = \frac{8\pi G}{c^2} \left( -\rho + \frac{3p}{c^2} + \frac{c^2}{4\pi G} \left( 3\frac{\square m}{m} - 2\Lambda \right) \right) \quad (16)$$

The cosmological constant  $\Lambda$  can be viewed as a field resulting from a perfect fluid with positive mass density  $\hat{\rho} = \Lambda c^2 / (8\pi G) = 6.38 \times 10^{-27} \text{ kg/m}^3$  pervading spacetime having an equation of state  $\hat{p} = -\hat{\rho} c^2 = -\Lambda c^4 / (8\pi G) = -5.73 \times 10^{-10} \text{ N/m}^2$  exerting negative pressure (tension) on spacetime over cosmological distances, such that  $-\hat{\rho} + 3\hat{p}/c^2 = -4\hat{\rho} = -2\Lambda c^2 / (4\pi G)$ . Notice that such a term has the gravitational constant  $G$  in the denominator to cancel the  $G$  factor multiplying the right hand side of the equation, because the cosmological constant is a field variable and not a concentrated mass source.

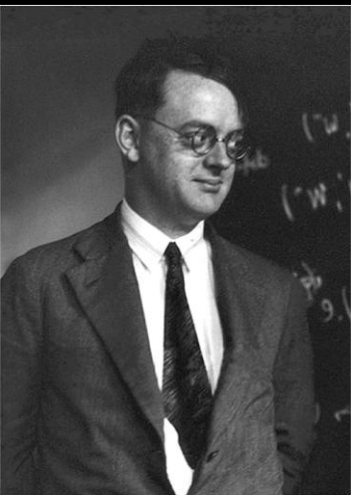
for the wave operator of the universe's fluid mass to be more significant than the cosmological constant term, it requires the wave operator of the universe's fluid mass to be  $\frac{\square m}{m} c^2 > \frac{2}{3} \Lambda c^2$  or, numerically,  $\frac{\square m}{m} c^2 > 7.13 \times 10^{-36} \frac{1}{s^2}$ .

Expanding the metric tensor in a powers series in terms of a small perturbation potential  $|h^{ik}| \ll 1$ , where we find that to first order:

$$g_{ik} = \eta_{ik} + h_{ik} + O(h^2) \quad (17)$$

perturbation  $h = h_{00} = -2\phi/c^2$ , where  $\phi$  is the Newtonian gravitational potential, therefore, substituting:

$$\begin{aligned} -M \square \phi = 4\pi G \left( -\rho + \frac{3p}{c^2} + \frac{c^2}{4\pi G} \left( 3 \frac{M \square m}{m} - 2\Lambda \right) \right) \\ -\frac{\partial^2 \phi}{c^2 \partial t^2} + \nabla^2 \phi = 4\pi G \left( \rho - \frac{3p}{c^2} + \frac{c^2}{4\pi G} \left( \frac{3}{m} \left( \frac{\partial^2 m}{c^2 \partial t^2} - \nabla^2 m \right) + 2\Lambda \right) \right) \end{aligned} \quad (22)$$





$$-\frac{\partial^2 \phi}{c^2 \partial t^2} + \nabla^2 \phi = 4\pi G \left( \rho + \frac{1}{4\pi G} \left( \frac{3}{m} \frac{\partial^2 m}{\partial t^2} + 2c^2 \Lambda \right) \right)$$

$m$  and  $G$  are related to each other, Eqs. (10) to (22) can be expressed solely in terms of  $G$  or  $m$ . One can express

$G = G_E + G_S$  where  $G_E$  is due to all the other masses in the universe and  $G_S$  is due to self-interaction from the particle's own mass field, and therefore  $G_S/G_E \ll 1$ . If

the energy of a mass body fluctuates with time, the gravitational  $G \approx G_E$  is constant and hence the fluctuation is entirely due to the self-interaction with its own-mass-field

then  $\frac{\partial^2 G}{\partial t^2} = \frac{\partial^2 (G_E + G_S)}{\partial t^2} = \frac{\partial^2 G_S}{\partial t^2}$ . In this sense, all the prior expressions can be interpreted in terms of constant  $G \approx G_E$

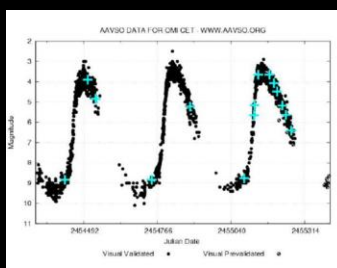
and  $\frac{\partial^2 m}{\partial t^2} = \frac{\partial^2 (m_E + m_S)}{\partial t^2} = \frac{\partial^2 m_S}{\partial t^2}$ . Alternatively, the equations

can be expressed solely in terms of  $G$  and its derivatives,

since  $\frac{\partial^2 m}{m \partial t^2} = -\frac{\partial^2 G}{2G \partial t^2} + \frac{3}{4} \left( \frac{\partial G}{G \partial t} \right)^2 = -\frac{\partial^2 G_S}{2G_E \partial t^2} + \frac{3}{4} \left( \frac{\partial G_S}{G_E \partial t} \right)^2$ .

Recently, Pitjeva and Pitjev (2013) gave a bound, based on observations of planets, within 95% probability, of  $-2.2 \times 10^{-21}/s < \frac{\partial G}{G \partial t} < 2.5 \times 10^{-21}/s$ .

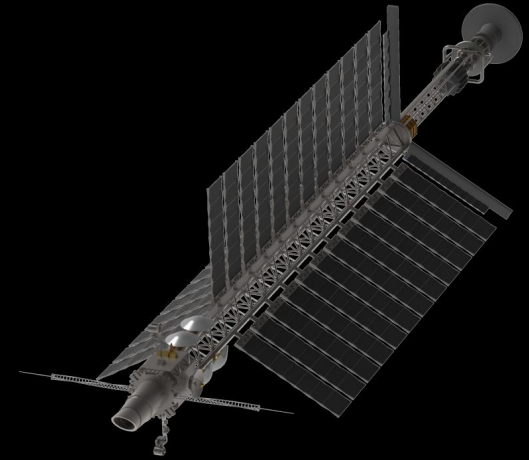
assume that  $G = G_E + G_S$  where  $G_E$  is constant and where  $G_S = G_s \sin[\omega t]$  varies harmonically, with  $G_E \gg G_s$  so that  $G \approx G_E$ , it follows that  $|\frac{\partial^2 G}{G \partial t^2}|_A = \frac{\omega_A^2}{\omega_B} |\frac{\partial G}{G \partial t}|_B$ . Then, inserting the data for the variable white dwarf G117-B15A: the bound  $|\frac{\partial G}{G \partial t}|_B \leq 1.30 \times 10^{-17}/s$  and frequency  $\omega_B = 2\pi/215.2s$ , and plugging-in the frequency for Woodward's experiment as  $\omega_A = 2\pi \times 35,000/s$  results in a much higher uncertainty bound of  $\frac{\partial^2 G}{G \partial t^2} \leq 2.15 \times 10^{-5}/s^2$  and  $0 \leq |\frac{\partial^2 m}{m \partial t^2}| \leq 1.08 \times 10^{-5}/s^2$ .



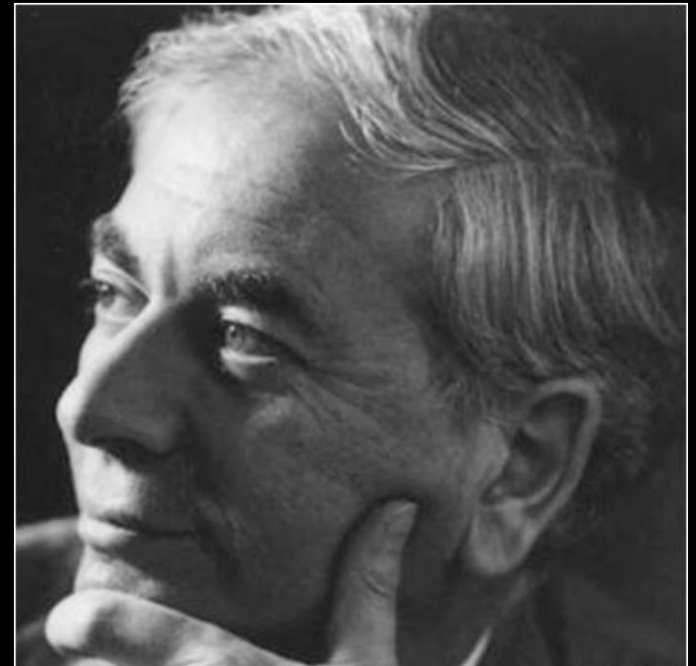
Rodal, 2017, "A Machian wave effect in conformal, scalar-tensor gravitational theory"



# Correct formulation



3. The correct terms can be obtained from Sciama using simple differentiation!





Sciama clearly distinguished between the *local* potential  $\phi$  ( $\sim 0$ ) and the total *universe's* potential  $\Phi$  ( $\sim -c^2$ ), and *used different notation for them*

Sciama, L., 1953, "On the Origin of Inertia," MNRAS, 113, 1, pp. 34-42

The total field at the particle is zero if

$$-\frac{M}{r^2} - \frac{\phi}{c^2} \frac{dv}{dt} = \frac{\Phi}{c^2} \frac{dv}{dt}$$

or

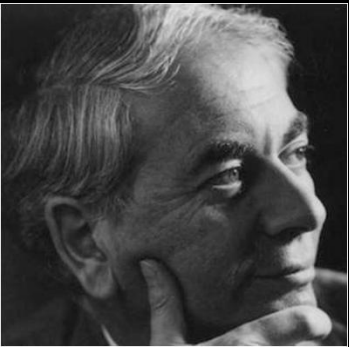
$$\frac{M}{r^2} = - \left( \frac{\Phi + \phi}{c^2} \right) \frac{dv}{dt}.$$

Furthermore, the gravitational constant satisfies the equation

$$\frac{\Phi + \phi}{c^2} = - \frac{1}{G}$$

or, since  $\phi \ll \Phi$  (*cf.* Section 4 (iii)),

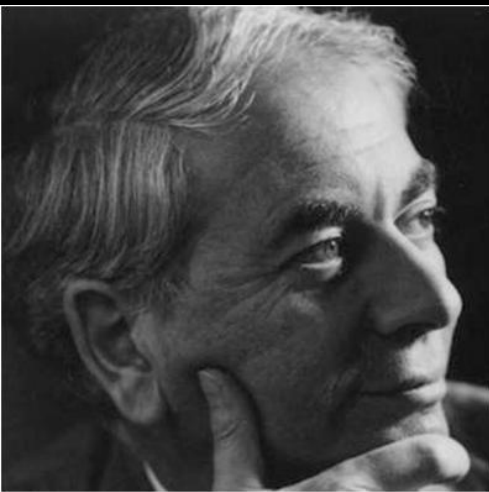
$$G\Phi = -c^2.$$



Sciama

Correctly calculate  
the 2<sup>nd</sup> time  
derivative of the  
total potential

Woodward/Fearn  
experiments can  
only fluctuate **local  
mass  $m_l$**



Gravitational potentials for the universe ( $\Phi$ ) and for the space region outside the external surface of a local mass ( $\phi$ ):

$$\Phi = -a G \frac{M_u}{R_u}; \quad \phi = -a G \frac{m_l}{r} \quad (1)$$

where  $a$  is a constant that can be  $\frac{3}{2}$ , 1, etc., depending on the method used to calculate the potential and on whether the potential solution is for the region inside a spherical mass or outside it. Using the overdot notation for time differentiation, for example:  $\ddot{\Phi} = \frac{\partial^2 \Phi}{\partial t^2}$  and since the mass  $M_u$  and radius  $R_u$  of the observable universe are constants (under short time duration) it follows that:

$$\begin{aligned} \ddot{\Phi} &= -a \frac{M_u}{R_u} \ddot{G} \\ &= \Phi \frac{\ddot{G}}{G} \end{aligned} \quad (2)$$

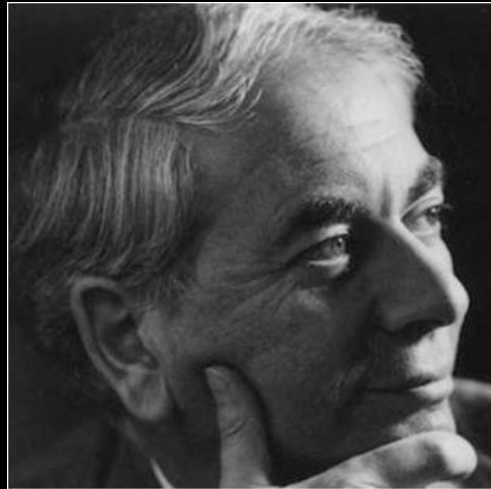
$$\ddot{\phi} = \phi \left( \frac{\ddot{G}}{G} + \frac{\ddot{m}_l}{m_l} - \frac{\ddot{r}}{r} + 2 \left( \frac{\dot{m}_l}{m_l} - \frac{\dot{r}}{r} \right) \left( \frac{\dot{G}}{G} - \frac{\dot{r}}{r} \right) \right)$$

$$\begin{aligned} \ddot{\Phi} + \ddot{\phi} &= (\Phi + \phi) \frac{\ddot{G}}{G} + \phi \left( \frac{\ddot{m}_l}{m_l} - \frac{\ddot{r}}{r} \right. \\ &\quad \left. + 2 \left( \frac{\dot{m}_l}{m_l} - \frac{\dot{r}}{r} \right) \left( \frac{\dot{G}}{G} - \frac{\dot{r}}{r} \right) \right) \end{aligned} \quad (3)$$

$$\approx \Phi \frac{\ddot{G}}{G} + \phi \left( \frac{\ddot{m}_l}{m_l} - \frac{\ddot{r}}{r} + 2 \left( \frac{\dot{m}_l}{m_l} - \frac{\dot{r}}{r} \right) \left( \frac{\dot{G}}{G} - \frac{\dot{r}}{r} \right) \right)$$

Correctly calculate  
the 2<sup>nd</sup> time  
derivative of the  
total potential

The solution for the  
interior region is  
similar. ***There is  
no singularity at  
the center of a  
uniform mass  
body.***



The inner solution matching the exterior for the inner (weak) gravitational potential  $\phi$  at radius  $r \leq r_o$  inside a ball with external radius  $r_o$ , and with total mass  $m_l$  resulting from a uniform mass density from  $r = 0$  to  $r = r_o$  is:

$$\phi = -a G \frac{m_l}{r_o} \left( \frac{3}{2} - \frac{r^2}{2r_o^2} \right) \quad (1)$$

where  $a$  is a constant that is typically 1. Therefore the potential at the center  $r = 0$  of the spherical mass is  $\phi_0 = -a \frac{3}{2} G \frac{m_l}{r_o}$  which is 50% larger in magnitude than the potential  $\phi_{r_o} = -a G \frac{m_l}{r_o} = \frac{2}{3} \phi_0$  at its surface  $r = r_o$ . Notice that the radial dependence of the potential changes from an inverse relationship  $-\frac{1}{r}$  outside the material ball to a quadratic dependence  $r^2$  inside the mass ball and that there is no singularity of the potential at the center of the spherical mass with uniform density. Using the overdot notation for time differentiation, for example:  $\ddot{\phi} = \frac{\partial^2 \phi}{\partial t^2}$  it follows that:

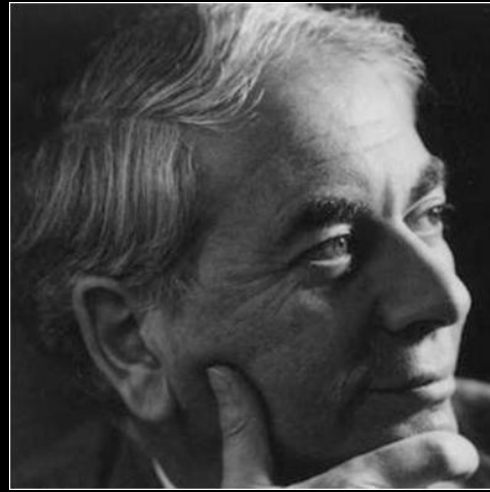
$$\begin{aligned} \ddot{\phi} = & \phi \left( \frac{\ddot{G}}{G} + \frac{\ddot{m}_l}{m_l} + 2 \frac{\dot{G}}{G} \frac{\dot{m}_l}{m_l} \right) \\ & - \phi_{r_o} \left( \frac{\ddot{r}}{r_o} \frac{r}{r_o} + \frac{\dot{r}}{r_o} \left( \frac{\dot{r}}{r_o} + 2 \frac{r}{r_o} \left( \frac{\dot{G}}{G} + \frac{\dot{m}_l}{m_l} \right) \right) \right) \end{aligned} \quad (2)$$

$$\begin{aligned} \ddot{\Phi} + \ddot{\phi} = & (\Phi + \phi) \frac{\ddot{G}}{G} + \phi \left( \frac{\ddot{m}_l}{m_l} + 2 \frac{\dot{G}}{G} \frac{\dot{m}_l}{m_l} \right) \\ & - \phi_{r_o} \left( \frac{\ddot{r}}{r_o} \frac{r}{r_o} + \frac{\dot{r}}{r_o} \left( \frac{\dot{r}}{r_o} + 2 \frac{r}{r_o} \left( \frac{\dot{G}}{G} + \frac{\dot{m}_l}{m_l} \right) \right) \right) \end{aligned} \quad (3)$$

$$\begin{aligned} \approx & \Phi \frac{\ddot{G}}{G} + \phi \left( \frac{\ddot{m}_l}{m_l} + 2 \frac{\dot{G}}{G} \frac{\dot{m}_l}{m_l} \right) \\ & - \phi_{r_o} \left( \frac{\ddot{r}}{r_o} \frac{r}{r_o} + \frac{\dot{r}}{r_o} \left( \frac{\dot{r}}{r_o} + 2 \frac{r}{r_o} \left( \frac{\dot{G}}{G} + \frac{\dot{m}_l}{m_l} \right) \right) \right) \end{aligned}$$



Correctly calculate the 2<sup>nd</sup> time derivative of the total potential



The solution at the center of the mass ball differs only by a factor of 3/2 from the one at the surface. There is no singularity at the center for a uniform mass body. **The local mass potential is infinitesimally small everywhere inside the MEGA drive.**

Rodal, 2017, "A Machian wave effect in conformal, scalar-tensor gravitational theory"

Let's calculate the value of the second time derivative of the local potential  $\ddot{\phi}$  at the external surface and at the center of the local mass. At the center  $r = 0$  of the ball:

$$\ddot{\phi}(r = 0) = \phi_{r_o} \left( \frac{3}{2} \left( \frac{\ddot{G}}{G} + \frac{\ddot{m}_l}{m_l} \right) + 3 \frac{\dot{G}}{G} \frac{\dot{m}_l}{m_l} - \left( \frac{\dot{r}}{r_o} \right)^2 \right) \quad (4)$$

The second time derivative of the potential at the surface  $r = r_o$  of the ball calculated using the solution valid for the interior of the ball is:

$$\ddot{\phi}(r = r_o) = \phi_{r_o} \left( \frac{\ddot{G}}{G} + \frac{\ddot{m}_l}{m_l} + 2 \frac{\dot{G}}{G} \frac{\dot{m}_l}{m_l} - \frac{\ddot{r}}{r_o} - \frac{\dot{r}}{r_o} \left( \frac{\dot{r}}{r_o} + 2 \left( \frac{\dot{G}}{G} + \frac{\dot{m}_l}{m_l} \right) \right) \right) \quad (5)$$

where, again, the potential at the ball's surface  $r = r_o$  is  $\phi_{r_o} = -a G \frac{m_l}{r_o} = \frac{2}{3} \phi_0$ . Notice that although the potential is continuous at the surface, the second time derivative of the potential, calculated at the surface of the ball from the interior solution, is equal to the solution calculated from the exterior of the ball, except for the square of the time derivative of the radius  $\left( \frac{\dot{r}}{r_o} \right)^2$  which term is discontinuous at the surface.

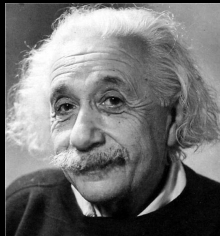
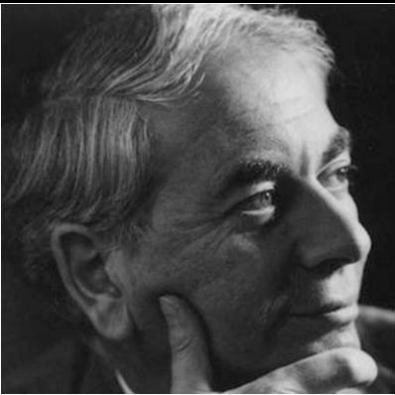
This discontinuity of  $\left( \frac{\dot{r}}{r_o} \right)^2$  at the surface of the ball is due to the discontinuity of the mass density at the surface (the mass density jumps from being zero on the exterior of the ball, to a finite constant value of mass density inside the ball).

The noteworthy point is that the prefactor multiplying the second time derivative of the local mass  $\frac{\ddot{m}_l}{m_l}$  is an infinitesimal (when divided by  $c^2$ ) quantity  $\phi_{r_o} = -a G \frac{m_l}{r_o} = \frac{2}{3} \phi_0$  (aside from the relatively unimportant difference between  $\frac{3}{2}$  if calculated at the center vs. 1 if calculated at the surface) whether one calculates the local potential at the surface or at the center of the uniform density ball. The fact that the local potential is infinitesimal (except for black holes) is in accord with all texts in general relativity.

$$\Phi/c^2 \sim -0.5$$

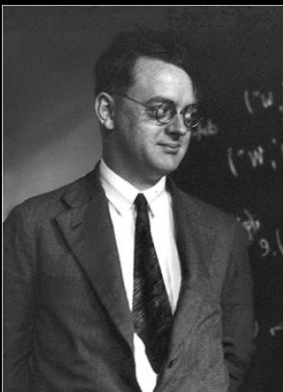
$$\phi/c^2 \sim 10^{-27}$$

$$\ddot{\Phi} + \ddot{\phi} \approx \Phi \frac{\ddot{G}}{G} + \phi \left( \frac{\ddot{m}_l}{m_l} - \frac{\ddot{r}}{r} + 2 \left( \frac{\dot{m}_l}{m_l} - \frac{\dot{r}}{r} \right) \left( \frac{\dot{G}}{G} - \frac{\dot{r}}{r} \right) \right)$$



Scalar-tensor theories  
(Hoyle Narlikar,  
Jordan-Brans-Dicke,  
string theory (dilaton),  
etc.) **term related to  $G$   
fluctuation**

**Infinitesimal** term  
related to **local mass  
fluctuation** present in  
General Relativity (3PN)  
(and Machian metric for  
the universe)



Ciufolini and Wheeler (1995) (p. 399)

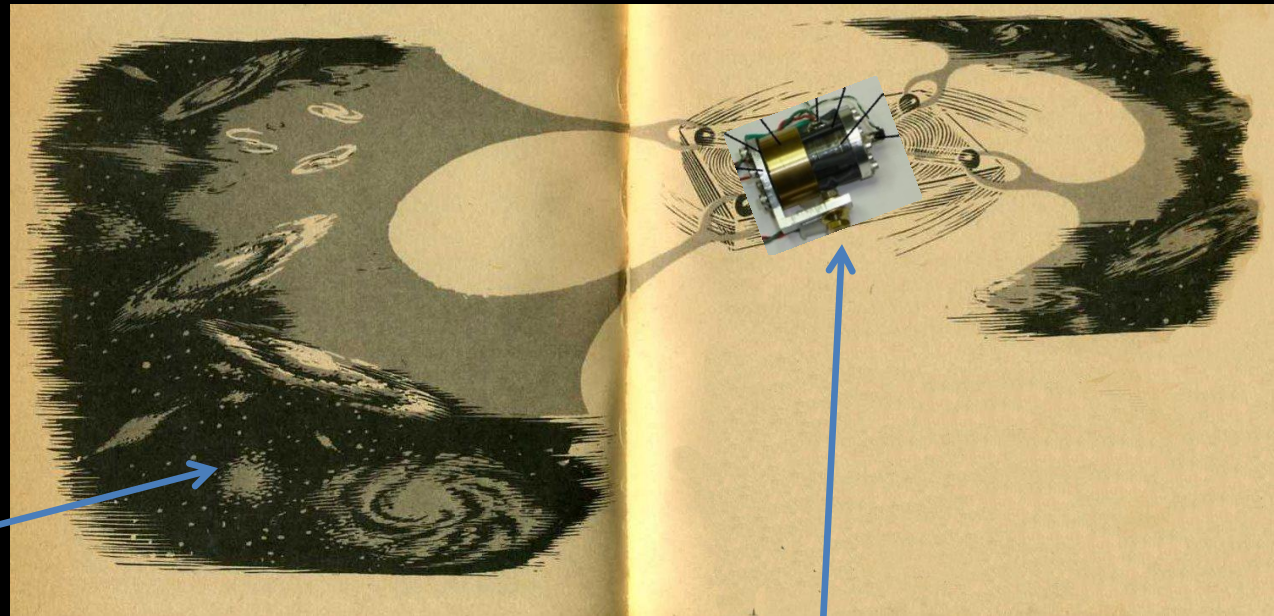
$$\Phi = - G M_u / R_u$$

Mass  $M_u = 10^{53} \text{ kg} =$

1000 kg

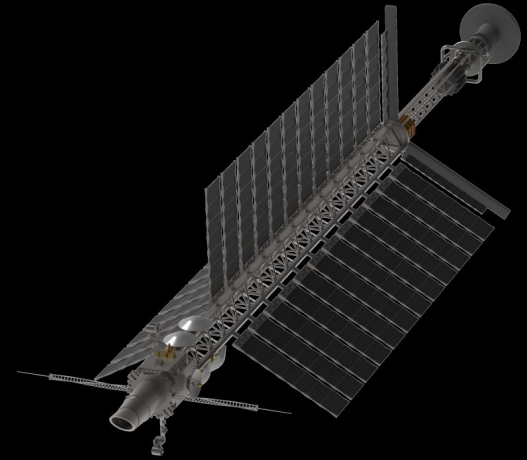
“there”

rules inertia  
here



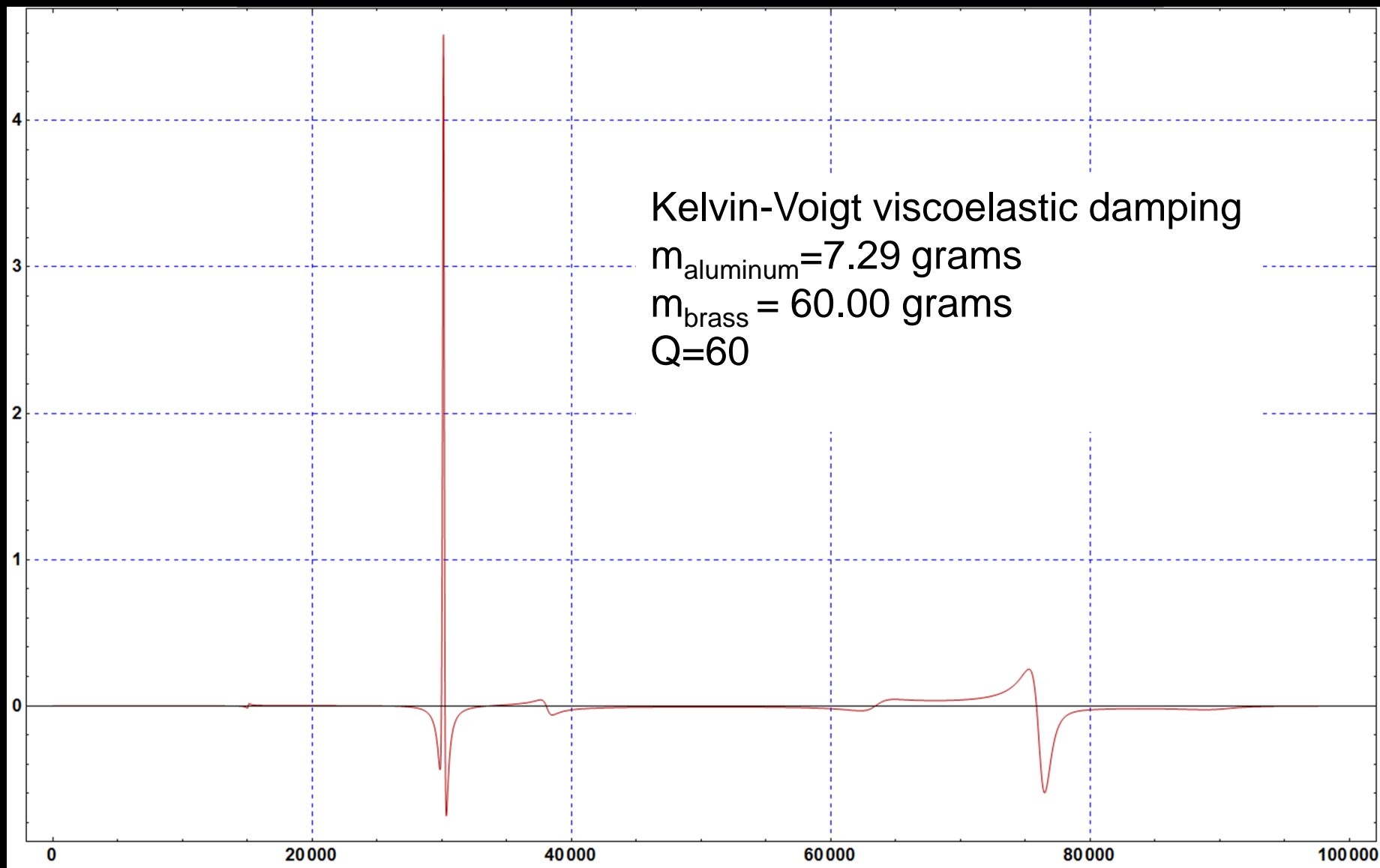
But you are only fluctuating the tiny mass  $m = 0.2 \text{ kg}$  *here, not there!*

# Exact solution results



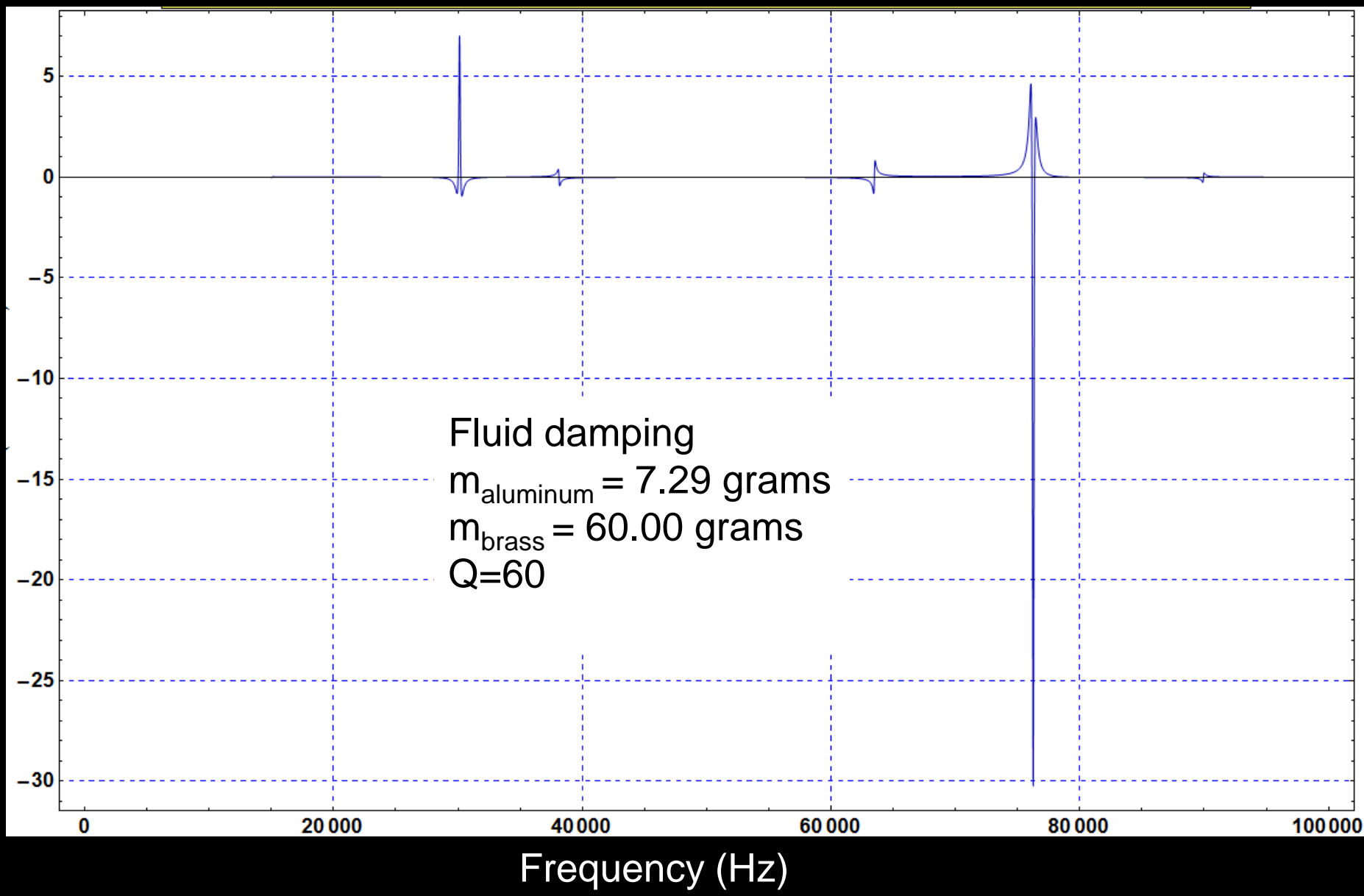


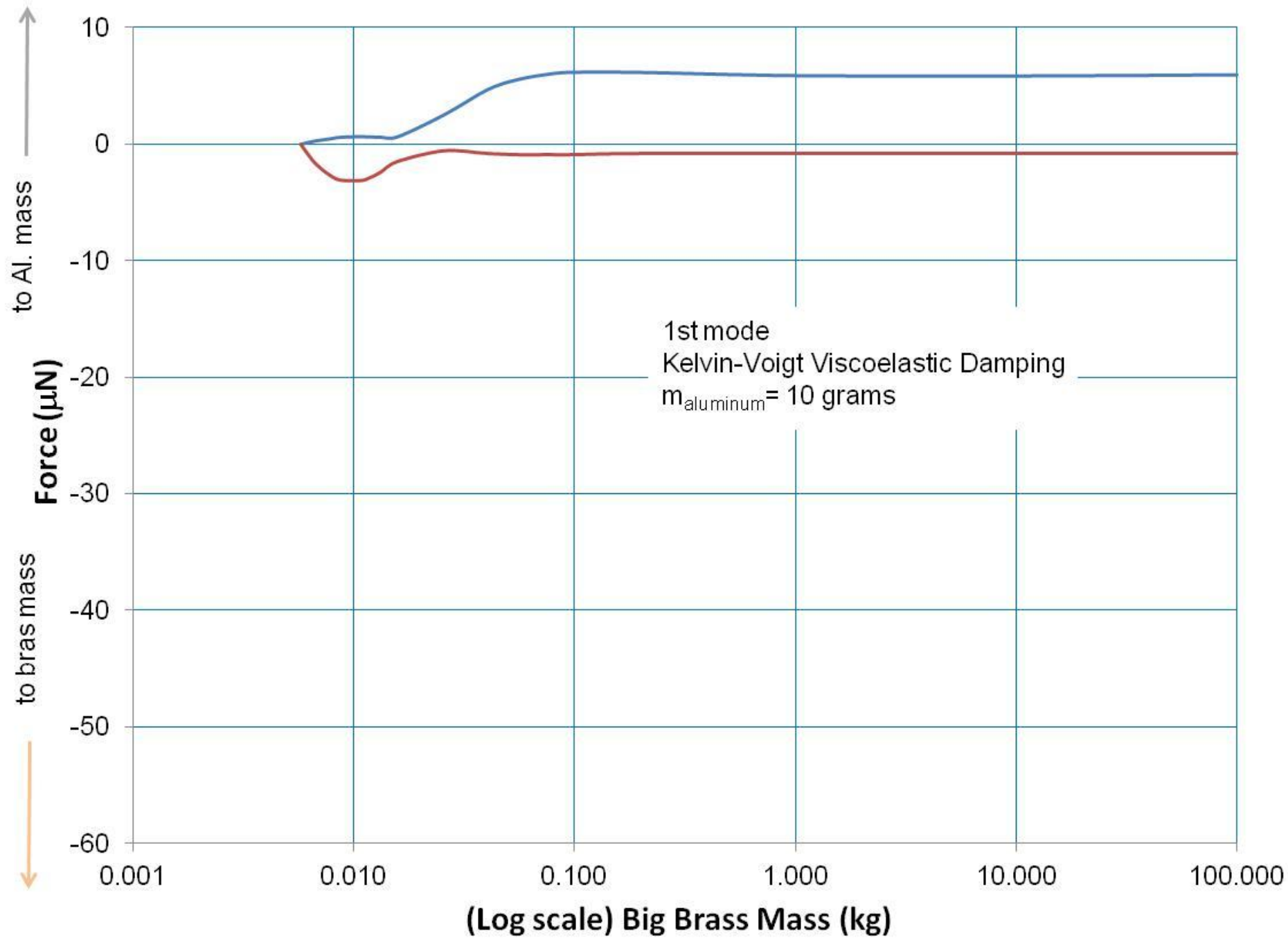
↑ to al. mass  
Force ( $\mu\text{N}$ )  
↓ to brass mass

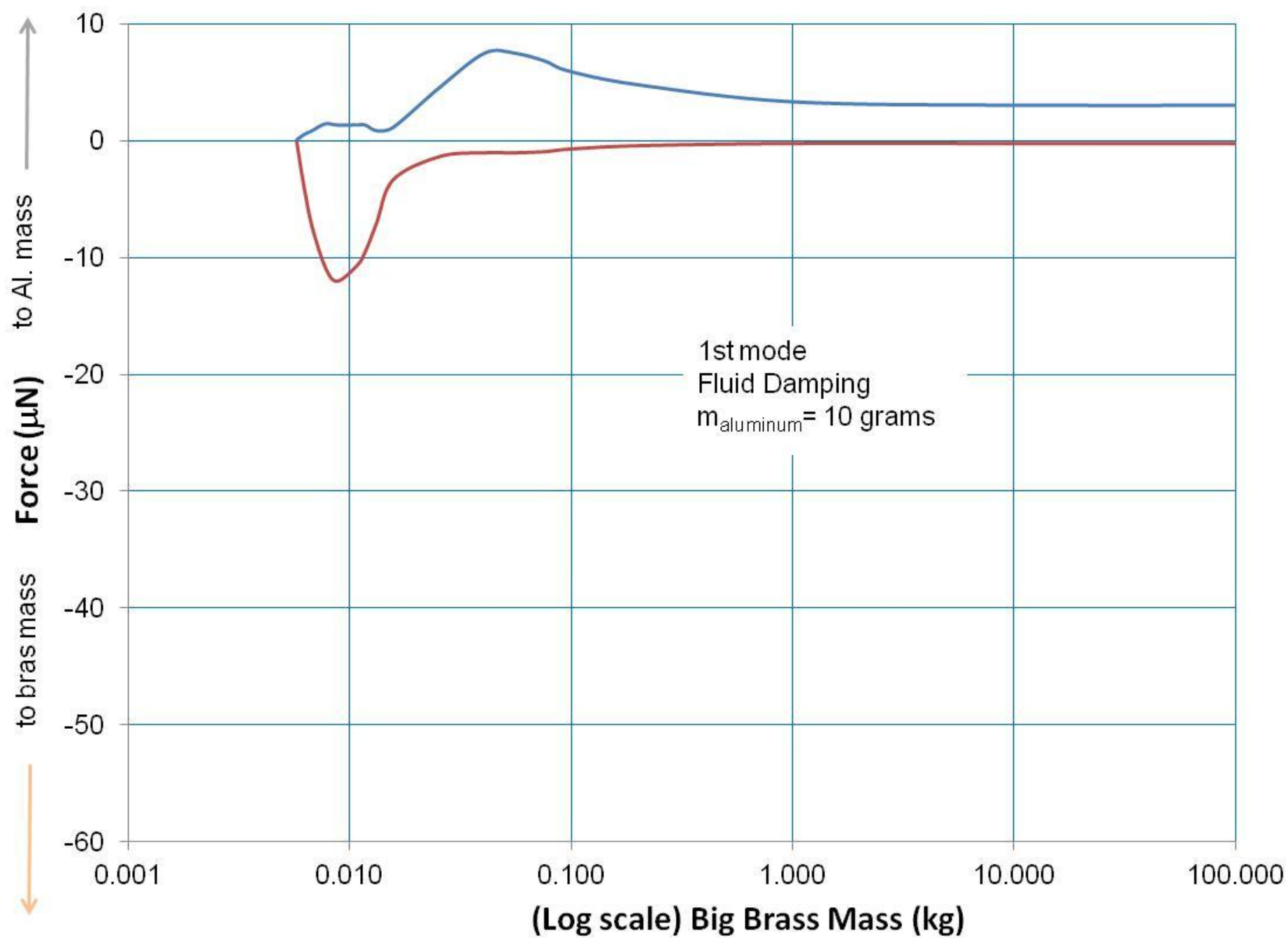


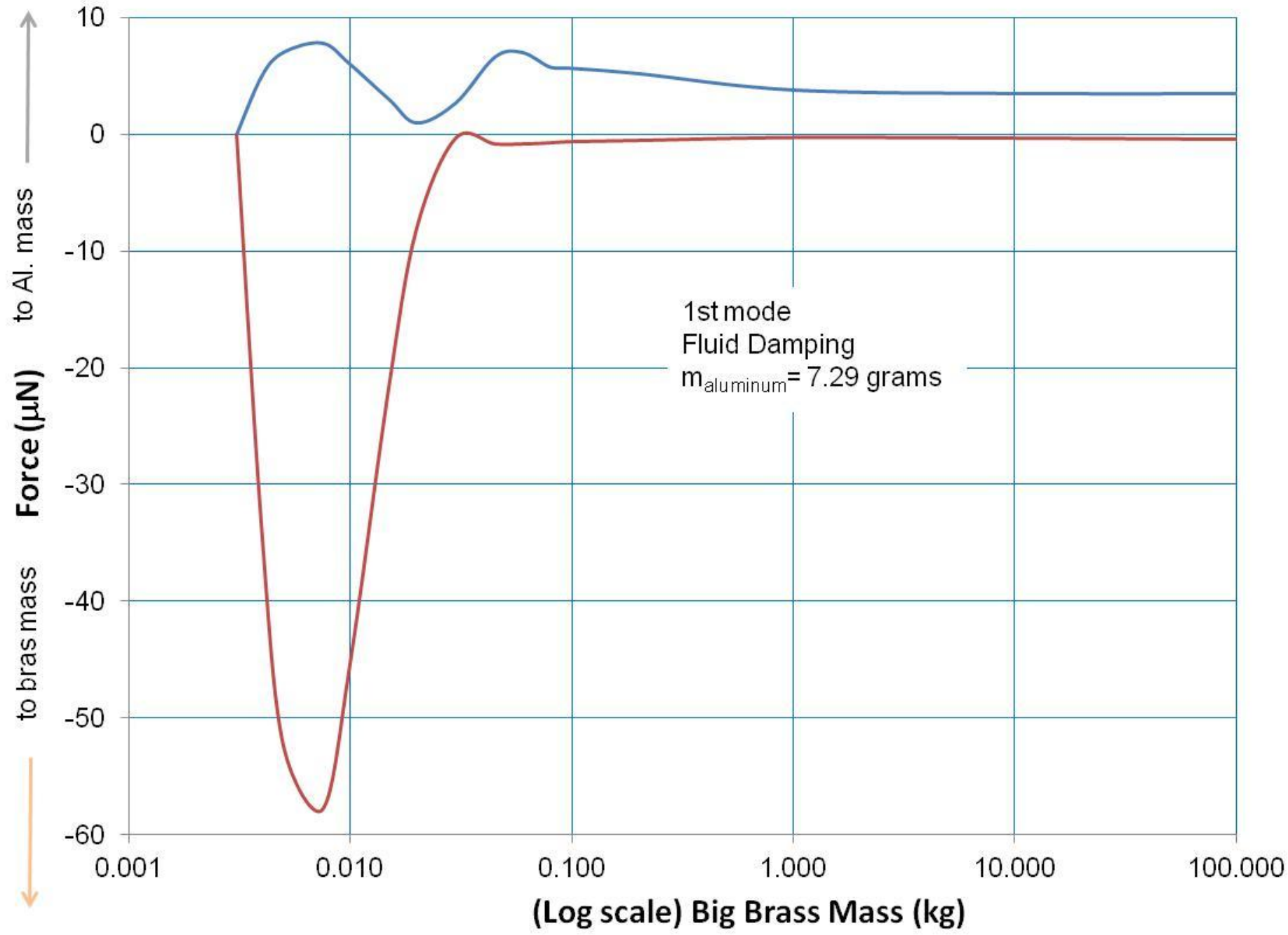
Frequency (Hz)

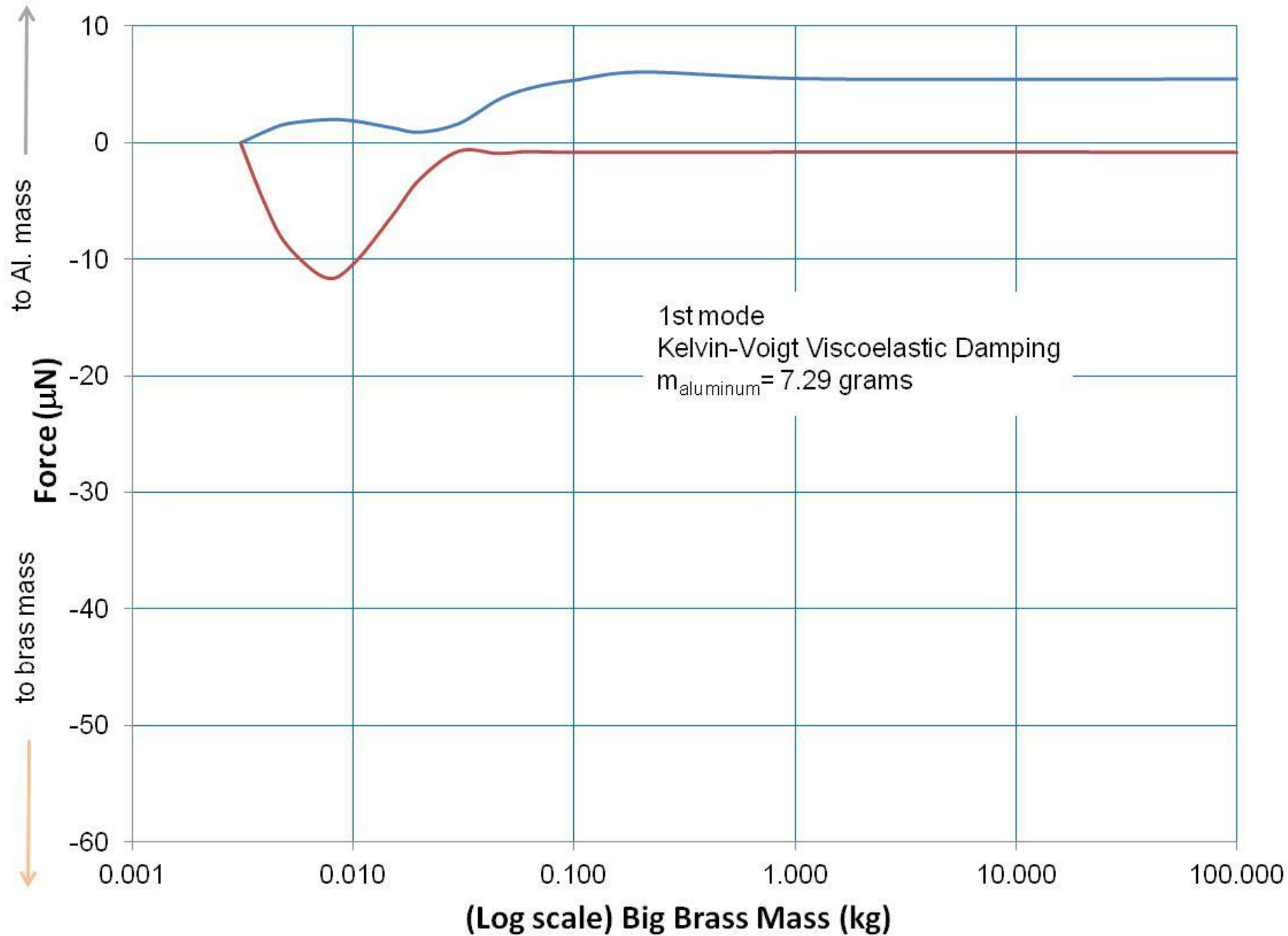
Force ( $\mu\text{N}$ )  
to al. mass  
to brass mass



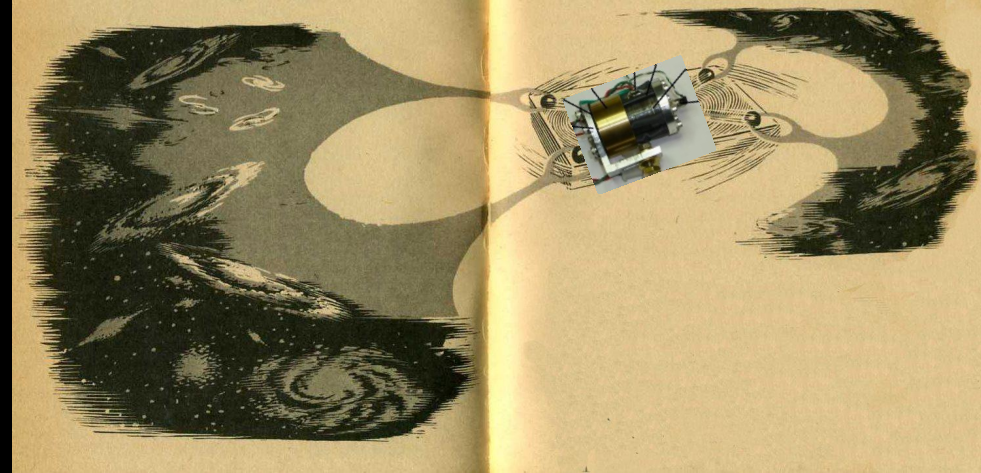








# Summary & conclusions



## • 9 confusions:

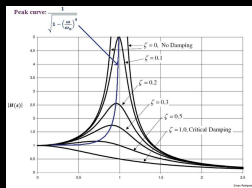
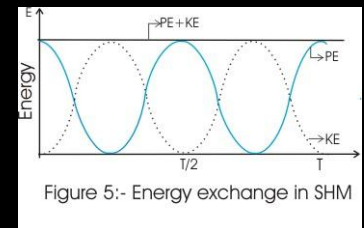
1. Confusing the *local* potential  $\phi$  ( $\sim 0$ ) with the *total universe's* potential  $\Phi$  ( $\sim c^2$ )

2.  $\nabla^2\phi - \frac{1}{c^2} \frac{\partial^2\phi}{\partial t^2} = 4\pi G\rho_0$  is not the Woodward effect

3. No valid 1-D model of Mach effect *space* propulsion

4. Can't have total energy fluctuation without damping

5. No sense in predicting frequency dependence when damping is neglected





# Summary & conclusions

- 9 confusions:

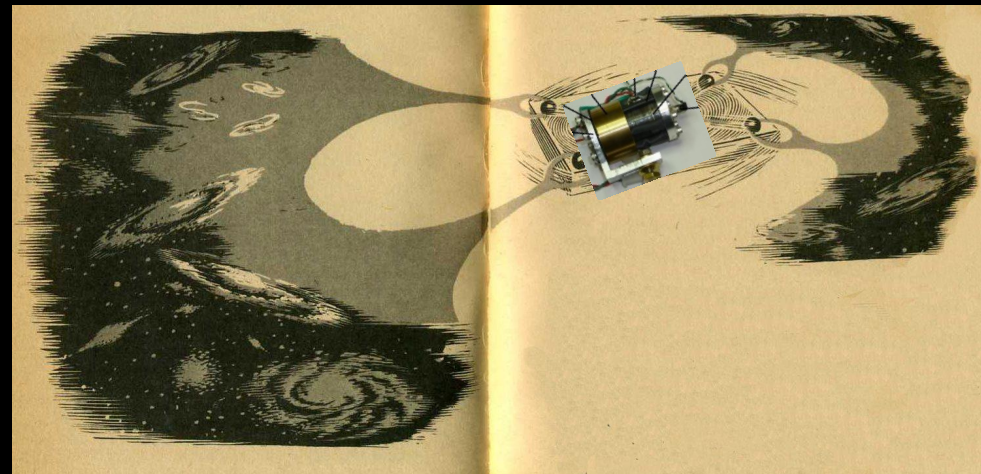
6. Magnitude of mass fluctuation has to be compatible with existing dynamic physical data

7. All energies gravitate: mechanical-energy is not the only energy with a gravitational potential

8. “m” in Hoyle-Narlikar is not a local mass source! It is a scalar field permeating all of spacetime

9. “mass-energy **there**  $M_u=10^{53}$  kg rules inertia here” ... but you are only fluctuating the tiny mass  **$m = 0.2$  kg here**

$$T^{\mu\nu} = \begin{bmatrix} \frac{1}{2} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) & S_x/c & S_y/c & S_z/c \\ S_x/c & -\sigma_{xx} & -\sigma_{xy} & -\sigma_{xz} \\ S_y/c & -\sigma_{yx} & -\sigma_{yy} & -\sigma_{yz} \\ S_z/c & -\sigma_{zx} & -\sigma_{zy} & -\sigma_{zz} \end{bmatrix}$$

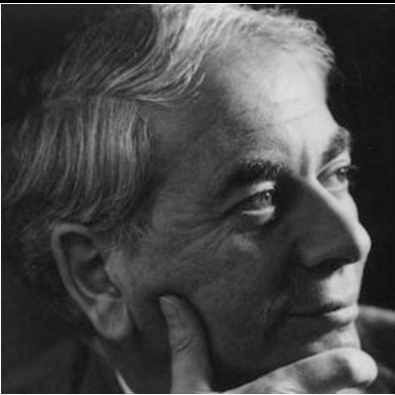
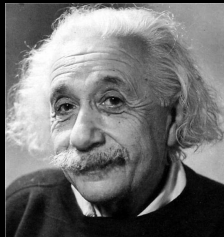



# Summary & conclusions

$$\Phi/c^2 \sim -0.5$$

$$\phi/c^2 \sim 10^{-27}$$

•Correct formulation:


$$\ddot{\Phi} + \ddot{\phi} \approx \Phi \frac{\ddot{G}}{G} + \phi \left( \frac{\ddot{m}_l}{m_l} - \frac{\ddot{r}}{r} + 2 \left( \frac{\dot{m}_l}{m_l} - \frac{\dot{r}}{r} \right) \left( \frac{\dot{G}}{G} - \frac{\dot{r}}{r} \right) \right)$$




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(*Hoyle Narlikar*,  
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***Infinitesimal*** term  
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General Relativity (3PN)  
(and Machian metric for  
the universe)

# Summary & conclusions

- Exact solution:

- partial differential equation for a continuous stack (infinite number of degrees of freedom): all eigenfrequencies and eigenmodes
- solution is very sensitive to
  - damping mechanism
  - mass distribution
- good agreement with scant data for: equal masses, and for brass= 65, 81, 97, 113 and 128 g. Need detailed tests particularly at lower brass mass for further verification.

# Propellantless space propulsion from a gravitational effect sourced by energy fluctuations

José J. A. Rodal, Ph.D.

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## Thank you for watching!

